TOKAMAK DESIGN AS ONE SUSTAINABLE SYSTEM

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Abstract: We derive the phenomena of Landau damping as a stationary point of entropy functions with Lagrangian methods. The steady states are described inside of some interval of numbers with infinite fuzzy logic controllers. The results are also true for local equilibriums, i.e. for some global non-equilibriums functions.

Key words: *Fuzzy entropy, Landau damping, real-time control, steady state, tokamak*

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1. Introduction

Charged particle motion in a magnetic field configuration has been proved theoretically to give rise to chaotic dynamics. There are many situations in which the existence of chaotic magnetic field lines has deep implications for the plasma confinement in tokamaks (the plasma wall-interactions, the control of disruptive instabilities).

Studies have been performed on the correlation between electrostatic and magnetic fluctuations. We need theoretical tools for describing the behavior of chaotic field lines in tokamak plasmas. This knowledge can be crucial for controlling the plasma stability when chaos is not desired.

The starting point of theoretical treatment of plasma confinement in tokamaks is the equilibrium configuration. Seen from the Hamiltonian point of view, any magnetostatic field that breaks the exact axisymmetric of the equilibrium tokamak field is a non-integrable perturbation. If the strength of these perturbations is not too high, it is possible to use standard result of Hamiltonian dynamics – like the KAM theory – to predict the behavior of the field lines in the presence of such almost-integrable magnetic systems.

Unperturbed flux surfaces will survive while others are destroyed, having in their place tubular shaped structures called magnetic islands. The KAM theory predicts that, for those irrational surfaces with safety factors sufficiently far from

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rational m/n, the topology is preserved and the surfaces are only slightly deformed from the unperturbed tori (KAM surfaces). Similar effects can be obtained by the theory of Landau damping.

The evolution of the poloidal flux in normalized cylindrical coordinates is given by the magnetic diffusion equation

$$\frac{\partial \psi}{\partial t} = \frac{\eta(T_e)}{\mu_0 \rho_b^2 F^2 \rho} \frac{\partial}{\partial \rho} (\rho F G H \frac{\partial \psi}{\partial \rho}) - R_0 H \eta(T_e) \frac{\langle j_{NI} B \rangle}{B_{\Phi,0}}, \tag{1.1}$$

where t is the time, ψ is the poloidal magnetic flux, η is the plasma resistivity, T_e is the plasma electron temperature, $\mu_0 = 4\pi \times 10^{-7} H/m$ is the vacuum permeability, j_{NI} is the non-inductive source of current density, B is the toroidal magnetic field and $\langle - \rangle$ denotes flux-surface average, F, G, H are geometric factors, which are functions of ρ [12].

The proposed closed-loop receding-horizon scheme shows potential for implementation in long-discharge tokamaks such as ITER. Future work towards reducing the computation time includes strategies such as: (i) implementation of model reduction techniques to approximate the infinite – dimensional PDE model by a low order finite – dimensional ODE model, (ii) approximation of the solution of the nonlinear control problem by successive computation of linear control problems, (iii) combination of off-line feed-forward and on-line feedback control strategies, where the feedback controller is intended to track an off-line-computed trajectory in the presence of disturbances and plant-model mismatches.

2. Real Time Control

As a first application of real-time density profiles, we performed feedback control of the shape of our profile. For this purpose, an actuator was required that is capable of modifying the spatial distribution of density. We have finally chosen central electron cyclotron heating as actuator.

For a given number of particles and a given, centrally peaked temperature profile, a fusion device yields higher fusion performance when the spatial distribution of the plasma density is peaked in the center. On the other hand, a too strongly peaked profile has negative effects for magneto-hydro-dynamical stability and central impurity accumulation. The reconstruction of the density profile is therefore an important step in the analysis of plasma discharges [10].

In the past few years, real-time diagnostics have been installed on many fusion devices. This allows for access to advanced operation regimes, which can only be established and sustained by feedback control. On ASDEX Upgrade, an equilibrium obtained by numerically solving the Grad-Shafranov equation is available for off-line data analysis, but could not yet be run in real-time during the last experimental campaign, as it is still consumed too much computing power.

To meet the operation parameters for data processing and control must be designed for both flexibility and performance, allowing easy integration of code from several developers and to guarantee the desired time cycle. In particular, a control cycle of 50 μs is needed for the vertical and horizontal positions control and a 500 μs loop is required for the plasma current, equilibrium and shaping control.

Although this approach has been successfully used in several tokamaks controlling on 50 μs cycles, it is not ITER-relevant given the long duration for the pulses, and the ability to reconfigure parameters during the discharge is required.

Efficient systems for plasma position stabilization form one of the key elements of tokamak operation, especially in case of increasing plasma performance. In order to create an algorithm for plasma position determination a suitable set of magnetic diagnostic coils must be selected. Selection was performed by computing the magnetic field in all magnetic diagnostic coils from modeled plasma with various parameters like plasma shape and plasma current profile. Control and calculation of plasma position has to be performed in real-time [6].

TEXTOR is the Tokamak Experiment for Technology Oriented Research dedicated to the field of plasma-wall interaction. A new real-time digital architecture has been installed for the purpose of plasma control. Two systems have already been successfully commissioned at TEXTOR. The first one is used to control the plasma shape (1 ms) and to calculate the plasma vertical position (20 μs). The second system is used to calculate in real-time the plasma density profile (10 ms), and the plasma horizontal position (20 μs) [9].

As part of the KSTAR discharge control system the plasma control system (PCS) provides real-time controllability, creating and sustaining plasma during the experimental campaign [4]. The master node, named kstarpcs, contains the main graphic user interface (GUI), application "Wave" communicates with the central controller (CCS) and assigns control input to the real-time node for every shot. The system has been used for simulation testing, poloidal field (PF) coil power supply commissioning and first plasma control. The KSTAR real-time plasma control system is based on a conceptual design and consists of a fast real-time computer communication cluster and software.

The equilibrium code solves the Grad-Shafranov equation:

$$\Delta^* \psi = -\mu_0 R j_\varphi \tag{2.1}$$

$$j_\varphi = -R\frac{dp}{d\psi} - \frac{f}{\mu_0 R}\frac{df}{d\psi}, \tag{2.2}$$

where ψ is the poloidal flux function, j_φ is the toroidal current density, p is the pressure, and $f = RB_\varphi$ is the toroidal field function.

It is important to check whether the target profile obtained from MHD stability analysis can be realized in terms of plasma transport. This will require a time-dependent transport simulation of the plasma profiles evolution with realistic models of anomalous transport coefficient, heating and current drive system, etc.

Measurements of integrated magnetic field from magnetic probes and flux loops and measurements of the currents flowing in the poloidal and toroidal field coils of ASDEX Upgrade make up the set of 100 measurements used as inputs to the tokamak equilibrium code. This code reconstructs the magnetic flux surfaces in ASDEX Upgrade and cannot be presently carried out in real time.

For the real-time control system, an algorithm of function parameters is used to calculate the magnetic equilibrium. For NTM stabilization, the mirror has to be set to the angle required to deposit Electron Cyclotron Current Drive (ECCD) power on the rational flux surface. An overview of the new data acquisition system for real-time magnetic flux surfaces and the control system of ASDEX Upgrade is shown.

3. Existence of Steady State

To begin let us consider a system which exchanges mass and energy with its environment. Let dS_i be the entropy production in the system due to irreversible processes and dS_e be the entropy flux due to exchanges between the system and the environment. The total entropy change in the system is given by $dS = dS_e + dS_i$. The second law states that $dS_i \geq 0$. However, if sufficient low entropy flux enters into the system then $dS_e \leq 0$, and it is possible that $\| dS_e \| \succ \| dS_i \|$, which implies that $dS < 0$. If this is the case then the system will be driven away from equilibrium. It is also possible for the system to eventually reach steady state and the accompanying coherent behavior for which Prigogine has developed a theory, for special cases.

$$\text{We have for } P = \frac{dS_i}{dt} \tag{3.1}$$

$$dP/dt < 0 \text{ away from steady-state} \tag{3.2}$$

$$dP/dt = 0 \text{ at a steady state.} \tag{3.3}$$

This is the famous Minimum Entropy Production rule which governs the behavior of dissipative structures in steady state. It can be easily shown that his rule ensures the stability of steady non-equilibrium states. It has now been found that this is only a sufficient condition, and not necessary. A general far-from-equilibrium thermodynamics and the theory of self-organization does not exist. In our approach this theory can be described inside of the concept of fuzzy entropies for deterministic and stochastic case.

In the analogy with Prigogine's non-equilibrium thermodynamics the so called universal evolution criterion of Glansdorff-Prigogine is formulated for the celebrated dynamical Lorentz system, generating the deterministic chaos. It is shown in detail numerically that such a criterion of Glansdorff-Prigogine is not universal in this sense.

Prigogine and coworkers have been attempting to develop a theory of non-equilibrium thermo-dynamical phenomena which would be valid far from full equilibrium, and which would apply both to the closed systems which asymptotically approach equilibrium and to systems subject to inputs or constraints such that these systems developed, asymptotically in time, steady states.

The paper [7] contains a regime of stable steady states for which the so-called entropy production cannot demonstrate that it is stable. This result strongly emphasizes that the Glansdorff-Prigogine criterion for stability is, at best, only a sufficient condition for stability, and so the violation of this criterion does not necessarily imply lack of stability.

For general non-equilibrium states, the examples show that time derivative of the second differential of entropy may be either positive or negative, and that a negative sign does not imply stability. The Glansdorf-Prigogine criterion for stability is based upon exhibiting a Liapunov function for the reaction. The methods of artificial intelligence can be applied for the cases of modern chaos theory.

Entropy, entropy time derivatives, or nonlinear entropy variations are inadequate to specify the probability of occupation of one stable or meta-stable state relative to another such state. The principle of minimum entropy generation is limited to the role of a frequently useful approximation, rather than being a basic physical principle [8]. It can be interpreted inside the theory of fuzzy control system.

Some steady-state systems, which permit more than one locally stable state, are such that we can learn how to compute the relative probability of occupation of such states. Steady state is frequently characterized as a state of minimum entropy production. This principle can only be expected to apply for steady states which are not too far from equilibrium. Without specifying the exact domain of validity of minimal entropy production we simply point out that it is not a universal principle.

The main objects of our study are dissipative dynamic systems which arise in various problems of kinetics. Non-equilibrium statistical physics is a collection of ideas and methods to extract slow invariant manifolds. From initial conditions the system goes quickly to a small neighborhood of the manifold, and after that moves slowly along this manifold. Selection of the slow (stable) positively invariant manifolds becomes an important problem [3].

Our collection of methods and algorithms can be incorporated into recently developed technologies of computer-aided multi-scale analysis which enable the "level jumping" between microscopic and macroscopic (system) levels.

The Kolmogorov spectrum is robust with respect to anisotropy of pumping. We can be sure that weak-turbulent Kolmogorov spectra are actually adequate for description of physical situations in situations where the kinetic equation actually applies. If the pumping is an external force (instead of instability), non-resonant spectra are just Kolmogorov- Arnold- Moser tori [13].

A different regime of turbulence occurs at higher levels of nonlinearity. It is tempting to identify frozen turbulence with the KAM regime in Hamiltonian systems of many degrees of freedom. This comparison can be made only with caution because many systems are not conservative and are pumped in a random way.

4. Landau Damping

The "standard model" of classical plasma physics is the Vlasov-Poisson-Fokker-Planck equation, here written with periodic boundary conditions and in dimensionless units:

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + F\left[f\right] \cdot \nabla_v f = \frac{\log \Lambda}{2\pi\Lambda} Q_L(f,f), \tag{4.1}$$

where $f = f(t,x,v)$ is the electron distribution function ($t \geq 0, v \in R^3, x \in T^3 = R^3/Z^3$)

$$F\left[f\right](t,x) = -\iint \nabla W(x-y) f(t,y,w)\,dwdy \tag{4.2}$$

is the self-induced force, $W(x) = 1/|x|$ is the Coulomb interaction potential, and Q_L is the Landau collision operator.

On very large time scales dissipative phenomena play a non-negligible role, and entropy increase is supposed to force the (slow) convergence to a Maxwellian distribution.

The nonlinear Landau damping for general interaction is obtained by C. Mouhot and C. Villani [11]. The unique solution of the nonlinear Vlasov equation satifies

$$\parallel \rho(t,\cdot) - \rho_\infty \parallel_{C^r(T^d)} \leq C \cdot \delta \cdot e^{-2\pi\lambda'|t|}, \tag{4.3}$$

where

$$\rho(t,x) = \int f(t,x,v)dv, \rho_\infty = \iint f_i(x,v)dvdx. \tag{4.4}$$

Furthermore, there are analytic profiles $f_{+\infty}(v)$, $f_{-\infty}(v)$ such that $f(t,\cdot) \overset{t\to\pm\infty}{\to} f_{\pm\infty}$ weakly $\int f(t,x,\cdot)dx \overset{t\to\pm\infty}{\to} f_{\pm\infty}$ strongly (in $C^r(R_v^d)$), these consequences being also $O(\delta e^{-2\pi\lambda'|t|})$. Large-time convergence is based on a reversible, purely deterministic mechanism. This result can be interpreted in the spirit of the KAM theorem: for the linear Vlasov equation convergence is forced by an infinity of invariant subspaces, which make the model "completely integrable"; as soon as one adds a nonlinear coupling, the invariance goes away but the convergence remains.

Let us define

$$\sigma(t,x) = \int_0^t \int_{R^d} F[f] \cdot \nabla_v \bar{f}(\tau, x - v\,|t-\tau|\,, v)dvd\tau. \tag{4.5}$$

It is the variation of density $\int f dv$ caused by the reaction of $\bar{f}$ on f. We show that if $\bar{f}$ has a high gliding regularity, then the regularity of σ in large time is better than what would be expected. The regularity of σ is better than that of $F[f]$.

The originality of the physical picture of Landau damping proposed gives results in agreement with sophisticated analytical analysis and Monte Carlo simulations. For times comparable with the inverse Landau damping rate the reversible Vlasov equation must be replaced by its irreversible linearized form for the perturbation, together with the quasilinear kinetic equation for the background plasma. Finally, it should go without mentioning that our considerations apply without restriction only to stable plasmas.

Since its first derivation in 1946, Landau damping has been the object of much discussion, aiming at understanding the "paradox" of an irreversible process being predicted by a collisionless model, namely Vlasov equation. We address the physics of Landau damping mechanism for the analysis of the steady state of externally excited waves which propagate and are absorbed in fusion plasmas. The key of argument is the observation that Vlasov equation is not an exact description of the dynamics of charged particles in the plasma, but only holds for times shorter than the collision relaxation time appropriate to the phenomena under consideration [1]. For slower phenomena, Fokker-Planck-Landau equation must be used. We describe a physical picture, which also retains the main feature of the transition to nonlinear Landau damping ($v_{coll} \ll v_{crit}$) on one side, and to the collisional regime typical for weakly ionized plasmas and gases ($v_{coll} \succ v$) on the other.

To understand the physical meaning of v_{crit}, we observe that the steady state solution of the Fokker-Planck equation emerges from a balance between the trapping of resonant particles in the wave electric field and collisions. The trapping in the wave field of amplitude E_0 tends to drive the distribution function away from the thermal equilibrium in the velocity interval

$$|v - v_{ph}| \prec \delta v \equiv (2eE_0/(mk))^{1/2}. \tag{4.6}$$

Because of the random walk nature of collision diffusion in velocity space, the characteristic detrapping time is

$$\tau_{\det r} = \tau_{coll}(v_{ph}) \left(\frac{\delta v}{v_{the}} \right)^2 = \tau_{coll} \left(\frac{v_{ph}}{v_{the}} \right)^3 \frac{e\varphi_0}{(m)v_{the}^2/2}. \tag{4.7}$$

If collisionality increases, the rate of detrapping increases, but the energy carried away by each particle decreases, and vice versa. The result of this irreversible process is Landau damping. Although collisions are essential to introduce irreversibility, the resulting damping rate is independent from the collision frequency. This is a universal property of resonant phenomena. The order introduced by the wave field (KAM surfaces) reduces diffusion in phase space.

Is Landau damping related to the more classical notion of stability in orbital sense? Orbital stability means that the system, slightly perturbed at initial time from an equilibrium distribution, will always remain close to this equilibrium. There is a widespread agreement that Landau damping cannot be hoped for if there is no orbital stability.

The equation is time-reversible, yet we are looking for an irreversible behavior at $t \to +\infty$ or $t \to -\infty$.

The value of entropy does not change in time which, physically speaking, means that there is no loss of information in the distribution function. In other words: damped solutions do exist, but do we ever reach them? In particular the force $F = -\nabla W * \rho$ converges exponentially fast to 0. The decay of the force field is the experimentally measurable phenomenon which may be called Landau damping.

The damping phenomenon is reinterpreted in terms of transfer of regularity between kinetic and spatial variables, rather than exchanges of energy; phase mixing is the driving mechanism. The analysis involves new families of analytic norms, measuring regularities in comparison with solutions of the free transport equation; new functional inequalities; a control of nonlinear echoes; sharp scattering estimates; and a Newton approximation scheme.

In contrast to models incorporating collisions, the Vlasov-Poisson equation is time-reversible. However, in 1946 Landau stunned the physical community by predicting an irreversible behavior on the basis of this equation. Landau concluded that the electric field decays exponentially fast.

There the thermodynamical formalism is used to compute the amount of heat Q which is dissipated when a (small) oscilating electric field $E(t,x) = Ee^{i(kx-\omega t)}$ (k a wave vector, $\omega \succ 0$ a frequency) is applied to a plasma whose distribution f^0 is homogeneous in space and isotropic in velocity space; the result is

$$Q = -\left|E\right|^2 \frac{\pi m e^2 \omega}{|k|\,|k|} \varphi'(\frac{\omega}{|k|}) \tag{4.8}$$

for $\varphi(v_1) = \int f^0(v_1, v_2, v_3) dv_2 dv_3$. In particular (4.8) is always positive, which means that the system reacts against the perturbation, and thus possesses some "active" stabilization mechanism.

5. Applications of Artificial Intelligence for Achieving a Path Toward Steady States

The conventional proportional-integral-derivative (PID) algorithm is still widely used in processing industries. The main reason is due to their simplicity of operation, ease of design, inexpensive maintenance, low cost, and effectiveness for most linear systems. In fact, 95% of control loops use PID and the majority is PI control.

However, its performance may degrade when applied to highly nonlinear processes. There is an important branch of adaptive control strategies using intelligent control algorithms (that includes neural and fuzzy approaches). Stochastic and heuristic optimization techniques such as evolutionary algorithms (EAs) have emerged as efficient tools for global optimization and have been applied to a number of numerical optimization problems in diverse fields in recent years. EAs use a population of structures (individuals) in which each one is a candidate solution for the optimization problem. Since they are population-based methods, they make a parallel search of the space of possible solutions, and are less susceptible to local minima. Design with neural compensation can be presented.

The neural network offers a framework for nonlinear compensation because they are simple topological structure and they can get a precise behavior in nonlinear dynamics optimizations. Learning of the radial basis function-neural network (RBF-NN) corresponds to determination of the centers t_i, variance (spread) σ, and the coefficients ω_i. Since RBN-NN networks are linear-in-the-parameters for fixed t_i and σ, the coefficients ω_i can be determined using the linear least-squares method. The choice of the values of t_i and ω_i is crucial for the performance of the neural compensation.

Optimization of controller design can be obtained using differential evolution (DE) approach. DE is based on a mutation operator, which adds an amount obtained by the difference of two randomly chosen individuals of the current population, in contrast to most EAs, in which the mutation operator is defined by a probability function.

We consider a probabilistic fuzzy logic (PFL) system composed of a set of following rules R_j : If x_1 is $X_1^h \ldots x_i$ is X_i^h, ... and x_l is X_l^h, then y is a_1 with a probability of success of ρ_{j1}, ... and a_n with a probability of success ρ_{jn}, where R_j is the jth rule of the rule base, X_i^h is the h-th linguistic value for input i, and $h \in \{1, 2, \ldots q_i\}$, where q_i is the total number of membership functions for the input x_i. Variable y denotes the output of the system, and a_k is a possible value for y, with $k \in \{1, 2, \ldots, n\}$ being the action number and n being the total number of possible actions that can be executed. The probability of this action to be successful is ρ_{jk}. These success probabilities ρ_{jk} are defined by

$$\rho_{jk}(t) = \frac{S\left[w_{jk}(t)\right]}{\sum_{k=1}^{n} S\left[w_{jk}(t)\right]}, \tag{5.1}$$

where S is an s-shaped function given by

$$S\left[w_{jk}(t)\right] = \frac{1}{1 - e^{-w_{jk}(t)}} \tag{5.2}$$

and $w_{jk}(t)$ is a real-valued weight that maps rule j with action k at time step t. Our learning method is based on the searching for stationary values of entropy functions.

Learning agents can tackle problems where preprogrammed solutions are difficult or impossible to design. The generalized probabilistic fuzzy reinforcement learning (RL) (GPFRL) method is a modified version of the actor-critic (AC) learning architecture, where uncertainty handling is enhanced by the introduction of a probabilistic term into the actor and critic learning, enabling the actor to effectively define an input-output mapping by learning the probabilities of success of performing each of the possible output actions.

In addition, the final output of the system is evaluated considering a weighted average of all possible actions and their probabilities [5]. Probabilistic modeling has proven to be a useful tool in many engineering fields to handle random uncertainties, such as in finance markets, astrophysics and in power systems.

For training the set of actions a_i, over trees of events, it is good to use recurrent neural networks (RNNs). Recently, a class of discrete-time RNNs, called echo state networks (ESNs) have been introduced, with the aim to reduce the complexity of computation encountered by standard RNNs. The principle behind ESNs is to separate the RNN architecture, called the "dynamical reservoir" or "hidden layer", and a memoryless output layer, called the "readout neuron". The recurrent architecture consists of a randomly generated group of hidden neurons with a specified degree of recurrent connections, and should satisfy the so-called "echo-state-property" to maintain stability [18].

The networks state at time instant k, denoted by $q(k)$ is a concatenation of the input $u(k)$, internal state $x(k)$ and delayed output $y(k-1)$, $q(k) = [u(k), \ldots, u(k-K+1), x_1(k), \ldots, x_N(k), y(k-1)]^T$ whereas the internal unit dynamics is described by $x(k) = f[W_{ip}u(k) + W_{in}x(k-1) + W_b y(k-1)]$, where $f(\cdot)$ is a vector-valued nonlinear activation function of the neurons within the reservoir. The echo state property is provided by randomly choosing an internal weight matrix and performing scaling to make the spectral radius $\rho(W_{in}) \prec 1$, thus ensuring that the network is stable. The ESN is trained based on the cost function $J(k) = {}^1\!/_2 \quad |e(k\ )| \cdot |e(k\ )| = {}^1\!/_2 e(k)e^*(k)$, where $e(k)$ is the instantaneous output error $e(k) = d(k) - y(k)$, and the $d(k)$ is the desired (teaching) signal. It can be shown that the maximum change in the cost function on the error surface occurs in the direction of the conjugate gradient.

We have our own method for obtaining one stable steady state or several stable steady states for different parameters. We use the technique of infinite fuzzy logic controllers for deterministic and stochastic case inside the description of kinetic phenomena. Let us assume that we deal with stochastic behavior. Controlled Markov process is given by the following elements: The sets of the countable state spaces x_i, $i = 0, 1, 2, \ldots$, the sets of the action spaces a_i, $i = 0, 1, 2, \ldots$, the probability distribution called the transition functions and the probability distributions called the initial distribution. Our aim is to find a control procedure under which the appropriate mathematical expectation of the appropriate path $L = x_0 a_0 x_1 a_1 \ldots x_n a_n$ is as large as possible. We have the next theorem.

Theorem. Let X be a Markov process with control and non-stationary strategy. Then, the stable steady state (or several such steady states) can be simulated approximately with infinite fuzzy logic controller by Bayesian learning rules.

Proof. We define the information entropy

$$S = -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} P_{ij}^{\Delta} \ln P_{ij}^{\Delta}. \tag{5.3}$$

The fuzzy distribution functions P_{ij}^{Δ} are considered as unknown variables still to be determined. The constraints $f(k,i)$ are measurements of some quantities and can be seen as expectation values. This is done by means of the requirement that (5.3) has a maximum value under given constraints

$$\sum_{j=0}^{\infty} P_{ij}^{\Delta} f_{ij}^{(k)} = f(k,i) \tag{5.4}$$

and

$$\sum_{j=0}^{\infty} P_{ij}^{\Delta} = 1 \tag{5.5}$$

for every time step i and index k. The maximization of (5.3) under the constraints (5.4) and (5.5) can be performed by the use of Lagrange multipliers λ_{ki} and $\lambda_i - 1$ (see [18]) for $S(i)$ is a finite sum over index j in the expression for entropy. We have that

$$\delta\left[S(i) - (\lambda_i - 1)\sum_j P_{ij}^{\Delta} - \sum_j\sum_k \lambda_{ki} P_{ij}^{\Delta} f_{ij}^{(k)}\right] = 0 \tag{5.6}$$

for every step i. Performing the variation of (5.6) by differentiating the bracket with respect to p_{ij}^{Δ} and putting the result equal to zero, we obtain

$$-\ln p_{ij}^{\Delta} - 1 - (\lambda_i - 1) - \sum_k \lambda_{ki} f_{ij}^{(k)} = 0 \tag{5.7}$$

Or equivalently $\ln p_{ij}^{\Delta} = -\lambda_i - \sum\limits_k \lambda_{ki} f_{ij}^{(k)}$ which, after putting both sides into the exponent of an exponential function, yields the required results

$$p_{ij}^{\Delta} = \exp\left[-\lambda_i - \sum_k \lambda_{ki} f_{ij}^{(k)}\right]. \tag{5.8}$$

It should be noted that we must still determine the Lagrange multipliers and that obtained probability depends crucially on the choice of constraints $f(k,i)$ which we chose since these, in turn, define the variable $f_{ij}^{(k)}$ in terms of which the probability distribution is expressed, what actually depends on appropriate actions. We arrive at maximum values of entropy or stationary values. If on each i-th step the optimal result is obtained by maximum entropy method, then the global maximum is obtained on the whole time scale. For controlled Markov chain we use the following infinite fuzzy logic controller with the Bayesian learning rule

$$\text{IF } P^{\Delta}(x_i) \text{ is } E_{1(i)} \text{ AND } P^{\Delta}(x_{i+1}/x_i) \text{ is } E_{2(i)} \text{ THEN } a_i \text{ is } U_{(i)}(i = 0, 1, 2 \ldots). \tag{5.9}$$

It can be interpreted as obtaining the steady states [14].

The proper object of a theory of turbulence is the study of ensembles of solutions, i.e. of collections of solutions with probability distributions that describe the frequency of their occurrence. We describe turbulence in terms of a suitable statistical equilibrium. As a simulation approach to study thermodynamics the Monte Carlo histogram technique is usually used. In the case of infinite degrees of freedom, obtaining the histogram corresponds to considering an appropriate probabilistic distribution. By our method, from the pictures of entropies we can derive the histograms of equilibriums or steady states.

For deterministic case, we must use the fuzzy Kolmogorov–Sinai entropy and apply the same method. But in this case the equilibrium is at the point of the minimum of the entropy [15]. Stability of the steady state could be obtained in terms of Liapunov's functionals [16], [17]. Same results could be applied to local equilibriums.

Conclusions

There are many situations in which the existence of chaotic magnetic field lines has deep implications for the plasma confinements in tokamaks. Besides active control of the density profile, there are different applications. A general far-from-equilibrium thermodynamics and the theory of self-organization does not exist. The nonlinear Landau damping for general interaction is obtained by C. Mouhot and C. Villani. For obtaining one or more stable states we used the method of infinite fuzzy logic controllers and fuzzy entropies. Possible applications of our theory could be found also for the problems of economy [2].

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