FROM QUANTUM TRANSFER FUNCTIONS TO COMPLEX QUANTUM CIRCUITS

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Abstract: The goal of the paper is to analyze the behavior of quantum systems which are connected in more complex circuits through serial, parallel or feedback ordering of various quantum subsystems. The Quantum State Transform (QST) is introduced to define a Quantum Transfer Function (QTF) that can be used to characterize behavior of complex circuits like e.g. stability better. It is shown that ordering more general quantum systems into feedback can yield to the definition of hierarchical quantum systems that are very close to well-known scale-free networks. Finally, all identified mathematical instruments are used to define quantum information/knowledge circuits as ordering of 2-port quantum subsystems covering both input/output information flow and content.

Key words: *Quantum transfer function, quantum state transform, inverse quantum state transform, quantum system ordering, complex quantum circuits, hierarchical quantum systems*

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1. Introduction

We can start with our previous results [1–3] where we tried to build knowledge about representations of quantum systems. We can continue in this way of thinking and start to analyze the connected quantum subsystems through serial, parallel or feedback ordering. We will find the assessment of behavior of complex connected systems with respect to their features.

First of all we summarize different representation of quantum systems. Matrix representation [4] is most general and obvious. The matrix representation can be simplified in case our system possesses some special features. If the additive principle of quantum state is fulfilled, we can introduce the Quantum State Transform (QST) and also the Quantum Transfer Function (QTF). The QTF has a strong analogy with the transfer function used in discrete signal and system theory. The QST is very similar to well-known z-transform [5]. The QTF can be used to analyze the behavior of connected systems where the poles position points out to stability/non-stability condition, etc.

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If our quantum system is general without additional features, the complex connection can yield into so called hierarchical quantum systems. Such a system possesses a lot of new features and it is similar to a scale-free network. The quantum link can be created through the quantum entanglement [3]. Generalization can yield into quantum swapping that can enlarge the links among nodes. Hierarchical networks can be generated either through quantum feedback or through passing through quantum non-linearity. Generally, the quantum links could be defined also among quantum hierarchical networks. We believe that such complex systems can store a large amount of information.

The all above-mentioned results can be applied to quantum information/knowledge circuits [12]. Each information/knowledge component is modeled as an input/output 2-port subsystem. Each port has its information flow and content. Ordering such information/knowledge components can define a more complex network which can be modeled also as 2-port with final input/output information flow/content.

The paper is structured as follows. Section 2 summarizes some variants of quantum system representations. Section 3 defines the rules for serial, parallel and feedback ordering in various representations. Section 4 introduces hierarchical quantum systems and their generation like feedback systems or passing through non-linear systems. Section 5 defines information/knowledge subsystems together with their ordering. These theories could be extended into ordering of information/knowledge subsystems. Section 6 concludes the paper.

2. Quantum Systems Representation

In this section, we analyze various representations of quantum systems. With respect to their special features, their representation can be simplified. For example a special feature called additive principle of quantum states can yield into the introduction of Quantum State Transform (QST) and then the Quantum Transfer Function (QTF). In case this feature is not fulfilled, we have to return to the well-known matrix representation of quantum systems.

2.1 Quantum State Transform (QST)

Theorem 1:

Let us define two N-dimensional quantum systems (qudits):

$$|\psi_1\rangle = \sum_{i=0}^{N} \alpha_i \cdot |\Phi_i\rangle, \;\; |\psi_2\rangle = \sum_{j=0}^{M} \beta_j \cdot |\Phi_j\rangle \tag{1}$$

together with the additive principle among indexes of combined quantum states $|\Phi_i\rangle, |\Phi_j\rangle$ as follows:

$$|\Phi_i\rangle\,|\Phi_j\rangle = |\Phi_{i+j}\rangle\,, \tag{2}$$

where i, j are integers[1] (positive or negative) and, for simplicity, suppose that

[1] This principle is obvious if we work with positive or negative quantum of energy. By scattering of two particles the energies are summed up (with the positive sign) or subtracted (positive and negative signs).

quantum states (1) are not dependent on time t.

Then the combined joint state can be expressed as:

$$|\psi_{1,2}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = \sum_{k=0}^{N+M} \gamma_k \cdot |\Phi_k\rangle, \tag{3}$$

where $\otimes$ means Kronecker product and γ_k is given:

$$\gamma_k = \sum_{i=0}^{N} \alpha_i \cdot \beta_{k-i} \quad . \tag{4}$$

Proof: The quantum combination of systems (1) is given by Kronecker product [4]:

$$|\psi_1\rangle \otimes |\psi_2\rangle = \sum_{i=0}^{N} \sum_{j=0}^{M} \alpha_i \cdot \beta_j \cdot |\Phi_i\rangle \otimes |\Phi_j\rangle. \tag{5}$$

If we apply the additive principle (2) together with the substitution $k = i + j$, we can define parameter γ_k given in (4).

Definition 1: Let us suppose the quantum system in Dirac notation:

$$|\psi\rangle = \alpha_0 \cdot |\Phi_0\rangle + \alpha_1 \cdot |\Phi_1\rangle + \cdots + \alpha_N \cdot |\Phi_N\rangle \,. \tag{6}$$

Quantum state transform $Q\,[.]$ and Inverse quantum state transform $Q^{-1}\,[.]$ of quantum system (6) can be defined as follows:

$$\begin{aligned} Q\,[\alpha_0 \cdot |\Phi_0\rangle + \alpha_1 \cdot |\Phi_1\rangle + \alpha_2 \cdot |\Phi_2\rangle + \cdots + \alpha_N \cdot |\Phi_N\rangle] &= \\ = \alpha_0 + \alpha_1 \cdot \eta + \alpha_2 \cdot \eta^2 + \cdots + \alpha_N \cdot \eta^N & \end{aligned} \tag{7}$$

$$\begin{aligned} Q^{-1}\left[\alpha_0 + \alpha_1 \cdot \eta + \alpha_2 \cdot \eta^2 + \cdots + \alpha_N \cdot \eta^N\right] &= \\ = \alpha_0 \cdot |\Phi_0\rangle + \alpha_1 \cdot |\Phi_1\rangle + \alpha_2 \cdot |\Phi_2\rangle + \cdots + \alpha_N \cdot |\Phi_N\rangle \,, & \end{aligned} \tag{8}$$

where $Q\,[.]$ is the complex function of complex variable η. Quantum state transform $Q\,[.]$ transforms the superposed quantum states into the polynomial function of variable η, and vice versa.

We can provide the QST of two quantum systems given in (1):

$$\begin{aligned} Q\,[|\psi_1\rangle] &= Q\left[\sum_{i=0}^{N} \alpha_i \cdot |\Phi_i\rangle\right] = \sum_{i=0}^{N} \alpha_i \cdot \eta^i, \\ Q\,[|\psi_2\rangle] &= Q\left[\sum_{j=0}^{M} \beta_j \cdot |\Phi_j\rangle\right] = \sum_{j=0}^{M} \beta_j \cdot \eta^j \end{aligned} \quad . \tag{9}$$

Then the QST of combined quantum systems (3) must be equal to:

$$Q\,[|\psi_1\rangle \otimes |\psi_2\rangle] = Q\left[\sum_{i=0}^{N} \alpha_i \cdot |\Phi_i\rangle\right] \cdot Q\left[\sum_{j=0}^{M} \beta_j \cdot |\Phi_j\rangle\right] = \sum_{k=0}^{N+M} \gamma_k \cdot \eta^k. \tag{10}$$

The features of QST seem to be analogical to the well-known z-transform [5]; however, the z-transform solves the evolution of discrete systems in time domain but the QST is an instrument for describing quantum systems in discrete quantum state space under the additive principle (2).

The time evolution in the z-transform is characterized by a function of complex variable z; the QST evolution by a complex function of variable η. In more general cases, the time dependent quantum system can be represented by a function of both complex variables z and η.

It is evident that the additive condition (2) must be fulfilled to achieve form (10). It will be later shown on the example of modeling of quantum hierarchical systems or networks that in more general cases the condition (2) does not have to be fulfilled.

2.2 Quantum Transfer Function (QTF)

We can imagine the quantum system with the input/output quantum states defined as follows:

$$|\psi_{IN}\rangle = \sum_{i=1}^{N} \alpha_i \cdot |\Phi_i\rangle, \;\; |\psi_{OUT}\rangle = \sum_{j=1}^{M} \beta_j \cdot |\Phi_j\rangle. \tag{11}$$

Let us suppose that our quantum system is characterized by an inner quantum state:

$$|\psi_{INNER}\rangle = \sum_{i=1}^{K} \gamma_i \cdot |\Phi_i\rangle. \tag{12}$$

Then the output state can be represented by a combination of input and inner states as follows:

$$|\psi_{OUT}\rangle = |\psi_{INNER}\rangle \otimes |\psi_{IN}\rangle\,. \tag{13}$$

The inner state should be understood as the quantum impulse function because it is equal to the system output $|\psi_{OUT}\rangle = |\psi_{INNER}\rangle$ in case the "quantum Dirac impulse" $|\psi_{IN}\rangle = 1 \cdot |0\rangle$ is applied on the input.

If the inner state (12) has a finite dimension (in our case K), we speak about the Quantum Finite Impulse Response (QFIR). On the other hand, for an infinite dimension the system is called the Quantum Infinite Impulse Response (QIIR).

With respect to the QST, the equation (13) can be rewritten:

$$Q\left[|\psi_{OUT}\rangle\right] = Q\left[|\psi_{INNER}\rangle\right] \cdot Q\left[|\psi_{IN}\rangle\right]. \tag{14}$$

The Quantum Transfer Function (QTF) can be defined as the ratio of the transformed QST output and input:

$$QTF\left(\eta\right) = \frac{Q\left[|\psi_{OUT}\rangle\right]}{Q\left[|\psi_{IN}\rangle\right]} = \frac{\sum_{j=0}^{M} \beta_j \cdot \eta^j}{\sum_{i=0}^{N} \alpha_i \cdot \eta^i}. \tag{15}$$

Under the condition (2), the nominator and denominator of QTF have polynomial forms. This principle is analogical to LTI (Linear Time Invariant) conditions in the z-transform. Of course, there exist quantum systems that do not fulfill the condition (2). In such a case, the QTF is a general function of η and in special cases it can be approximated by the polynomial form given in (15) through e.g. Padé approximation [6].

2.3 Interpretation of Quantum Transfer Function (QTF)

In this section, we ask what features can be identified from the available quantum system described by the QTF. We suppose the quantum system has an infinite impulse response (QIIR) if the QTF denominator exists.

The QIIR systems given by (15) can be divided in stable or unstable parts according to the poles position. The general QTF can be rewritten in the following form:

$$QTF\left(\eta\right)=\frac{\left(\eta-\tilde{\eta}_0\right)\ldots\left(\eta-\tilde{\eta}_M\right)}{\left(\eta-\hat{\eta}_0\right)\cdot\ldots\cdot\left(\eta-\hat{\eta}_N\right)}=\frac{k_0}{\left(\eta-\hat{\eta}_0\right)}+\cdots+\frac{k_N}{\left(\eta-\hat{\eta}_N\right)},\tag{16}$$

where $\tilde{\eta}_0,\ldots,\tilde{\eta}_M$ are nulls, $\hat{\eta}_0,\ldots,\hat{\eta}_N$ poles, and $k_0,\ldots,k_N$ constants (all nulls, poles or constants can be complex numbers). In analogy to the z-transform, equation (16) can be rewritten as:

$$QTF\left(\eta^{-1}\right)=\frac{k_0\cdot\eta^{-1}}{\left(1-\hat{\eta}_0\cdot\eta^{-1}\right)}+\cdots+\frac{k_N\cdot\eta^{-1}}{\left(1-\hat{\eta}_N\cdot\eta^{-1}\right)},\tag{17}$$

where η^{-1} points out to the last index of quantum state.

Each x-part $x\in\{0,1,...,N\}$ of sum (17) represents the generator (difference equation) of x-output parameter ${}^x\beta_j$ of output quantum state $|\Phi_j\rangle$:

$${}^x\beta_j=\hat{\eta}_x\cdot{}^x\beta_{j-1}+k_x\cdot\alpha_{i-1}\tag{18}$$

$$\beta_j={}^0\beta_j+{}^1\beta_j+\cdots+{}^N\beta_j.\tag{19}$$

From (18) and (19) it is evident that all x-parts are parallelly ordered and the stability condition of quantum system (17) has to be defined for all poles $x\in\{0,1,\ldots,N\}$:

$$|\hat{\eta}_x|<1.\tag{20}$$

For *stable quantum systems,* the normalization condition is finite and there exist real probabilities assigned to different superposed quantum states.

For *unstable quantum systems* (at least one pole not fulfilling condition (20)), the normalization condition is infinite (the sum of modulus of complex parameters of all superposed quantum states) and all complex parameters of the output quantum system $|\psi_{OUT}\rangle$ are infinite. Such a system cannot give us reasonable measured values until it is passed through another quantum system whose nulls can remove the unstable poles of QTF.

The question is where in real life this kind of system exists and how to use this feature. What will happen if the information is modulated on different states e.g. by phase modulation but the states are unstable? Could such information be available by measurements? Do the poles represent something like *quantum information black holes*?

The next question is how such a system behaves *on the boundary of stability/non-stability* area. On stability limit, there exists an infinite number of equally probable superposed states where the probability of each state is, due to normalization condition, extremely low (in limit it is zero). This principle can be interpreted as a way how to create the empty space in which only basic rules and principles like quantization, additive principle, etc. are incorporated (we can call it *quantum information vacuum*). How can the entanglement change the behavior of system close to the boundary limit of quantum stability/non-stability? Can we, with the help of entanglement, code the information in the quantum information vacuum system beforehand? Such entangled information should change the behavior of future particles added into the system and influence the quantum system evolution.

Maybe this approach of generation of empty quantum space affected by the entanglement can be extended to more complex spaces and we could define the quantum information vacuum system with our physical laws.

2.4 Matrix representation of quantum systems

A general representation of quantum systems can be given in the matrix form (transform of complex coefficients assigned in quantum states):

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ .. \\ \beta_N \end{bmatrix}_{OUT} = A \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_N \end{bmatrix}_{IN}, \tag{21}$$

where matrix A represents our quantum system (its inner features), $[.]_{IN}$ the vector of input complex quantum parameters, and $[.]_{OUT}$ the vector of output quantum parameters as it was given in (11) under assumption $N = M$. In the form (21), condition (2) is not fulfilled.

The matrix A should be unitary, that means it should have the following feature:

$$A^{-1} = A^{+}, \tag{22}$$

where the symbol + stands for Hermitian conjugation, which combines both transposition and complex conjugation [4].

The matrix form can be generally extended as described in [7], where the complex quasi-ergodic quantum models were introduced.

3. Serial, Parallel and Feedback Ordering of Quantum Systems

3.1 Ordering of quantum systems in the matrix form

There exist two quantum systems $S_1 : (IN,1 \rightarrow OUT,1)$ and $S_2 : (IN,2 \rightarrow OUT,2)$ expressed as follows:

$$|\Phi_{IN,1}\rangle = \sum_{i=0}^{N_1} {}^{IN,1}\alpha_i \cdot \left|{}^{IN,1}\Phi_i\right\rangle, \;\; |\Phi_{OUT,1}\rangle = \sum_{j=0}^{M_1} {}^{OUT,1}\beta_j \cdot \left|{}^{OUT,1}\Phi_j\right\rangle \tag{23}$$

$$|\Phi_{IN,2}\rangle = \sum_{i=0}^{N_2} {}^{IN,2}\alpha_i \cdot \left|{}^{IN,2}\Phi_i\right\rangle, \ \ |\Phi_{OUT,2}\rangle = \sum_{j=0}^{M_2} {}^{OUT,2}\beta_j \cdot \left|{}^{OUT,2}\Phi_j\right\rangle, \tag{24}$$

where ${}^{IN,1}\alpha_i, {}^{OUT,1}\beta_j, {}^{IN,2}\alpha_i, {}^{OUT,2}\beta_j$ are inputs/outputs complex parameters with the transitions matrices A, B given by:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ .. \\ \beta_{M_1} \end{bmatrix}_{OUT,1} = \begin{bmatrix} A_{1,1} & A_{1,2} & . & A_{1,N_1} \\ . & . & & . \\ . & & . & . \\ A_{M_1,1} & . & . & A_{M_1,N_1} \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_{N_1} \end{bmatrix}_{IN,1} \tag{25}$$

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ .. \\ \beta_{M_2} \end{bmatrix}_{OUT,2} = \begin{bmatrix} B_{1,1} & B_{1,2} & . & B_{1,N_2} \\ . & . & & . \\ . & & . & . \\ B_{M_2,1} & . & . & B_{M_2,N_2} \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_{N_2} \end{bmatrix}_{IN,2} . \tag{26}$$

By using Dirac notation, the quantum system can be described as follows:

$$\begin{aligned} |\Phi_{OUT,1}\rangle &= \sum_{j=0}^{M_1} {}^{OUT,1}\beta_j \cdot \left|{}^{OUT,1}\Phi_j\right\rangle = \\ &= \sum_{j=0}^{M_1} \left(A_{j,1} \cdot \alpha_1 \cdot \left|{}^{IN,1}\Phi_j\right\rangle + \cdots + A_{j,N_1} \cdot \alpha_{N_1} \cdot \left|{}^{IN,1}\Phi_j\right\rangle\right). \end{aligned} \tag{27}$$

Equation (27) can be interpreted as the *conditional quantum probability.* The superposed input state:

$$A_{j,1} \cdot \alpha_1 \cdot \left|{}^{IN,1}\Phi_j\right\rangle + \cdots + A_{j,N_1} \cdot \alpha_{N_1} \cdot \left|{}^{IN,1}\Phi_j\right\rangle \tag{28}$$

can happen only if the output state $\left|{}^{OUT,1}\Phi_j\right\rangle$ was first selected/measured. It means that quantum state (28) is conditioned by quantum state $\left|{}^{OUT,1}\Phi_j\right\rangle$.

Let us start with introducing the *serial ordering* of quantum systems, meaning that $OUT, 1$ is connected to the input $IN, 2$ under the condition $M_1 = N_2$. The transition matrix between the input vector of system S_1 and the output vector of system S_2 is given:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ .. \\ \beta_N \end{bmatrix}_{OUT,2} = B \cdot A \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_N \end{bmatrix}_{IN,1} . \tag{29}$$

For the *parallel ordering* of quantum systems we suppose that $IN, 1$ and $IN, 2$ are equal under the condition$N_1 = N_2$. The transition matrix between the common input vector of systems S_1 and S_2 and the output vector given by the sum of output vectors of both systems S_1 and S_2 is given:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ .. \\ \beta_N \end{bmatrix}_{OUT,2} = (B + A) \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_N \end{bmatrix}_{IN,1} = (B + A) \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_N \end{bmatrix}_{IN,2} . \tag{30}$$

Let us introduce the *feedback ordering* of quantum systems meaning that the quantum system S_2 (24) is connected to the positive/negative feedback to the quantum system S_1 (23). For simplicity, we suppose the condition of the same dimensionalities $M_1 = M_2 = N_1 = N_2 = N$. The transition matrix between the input and output complex vectors (we use IN, OUT notation for the whole feedback circuit) is given:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ .. \\ \beta_N \end{bmatrix}_{OUT} = A \cdot (1 \mp B \cdot A)^{-1} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_N \end{bmatrix}_{IN} , \tag{31}$$

where 1 means the unit matrix and $(.)^{-1}$ the inverse matrix operation.

It can be easily analyzed that such circuits can fall into the unstable state or otherwise, due to its features, the unstable quantum system can be stabilized. The feedback ordering of quantum systems with unitary matrices A, B can yield into the quantum system with the general non-unitary transition matrix given in (31).

3.2 Ordering of quantum systems in the QTF form

In this section, we suppose to have available two quantum systems defined through the QTF under the condition (2):

$$Q_1\left(\eta^{-1}\right) = \frac{Q\left[|\psi_{OUT,1}\rangle\right]}{Q\left[|\psi_{IN,1}\rangle\right]} = \frac{\sum\limits_{j=0}^{M} {}^{1}\beta_j \cdot \eta^{-j}}{\sum\limits_{i=0}^{N} {}^{1}\alpha_i \cdot \eta^{-i}} \tag{32}$$

$$Q_2\left(\eta^{-1}\right) = \frac{Q\left[|\psi_{OUT,2}\rangle\right]}{Q\left[|\psi_{IN,2}\rangle\right]} = \frac{\sum\limits_{j=0}^{M} {}^{2}\beta_j \cdot \eta^{-j}}{\sum\limits_{i=0}^{N} {}^{2}\alpha_i \cdot \eta^{-i}}. \tag{33}$$

The *serial ordering* is given as:

$$Q\left(\eta^{-1}\right) = Q_1\left(\eta^{-1}\right) \cdot Q_2\left(\eta^{-1}\right). \tag{34}$$

The *parallel ordering* can be expressed in the form:

$$Q\left(\eta^{-1}\right) = Q_1\left(\eta^{-1}\right) + Q_2\left(\eta^{-1}\right), \tag{35}$$

and for the *feedback ordering*, the QTF can be written:

$$Q\left(\eta^{-1}\right) = \frac{Q_1\left(\eta^{-1}\right)}{1 \mp Q_1\left(\eta^{-1}\right) \cdot Q_2\left(\eta^{-1}\right)}. \tag{36}$$

4. Hierarchical Quantum Systems

In the past decade, much effort has been made the area of complex networks [8]. In the *random network nodes*, we have approximately the same number of links, making the distribution of connectivity homogeneous. In the *scale-free network*, contribution of hubs or highly connected nodes to overall connectivity is dominating. $\mathrm{P}(k)$ is connectivity distribution [9], defined as the probability that a randomly chosen node in a network has exactly k links. For random networks $\mathrm{P}(k)$, Poisson distribution follows. In free-scale networks, a relatively small number of hubs $\mathrm{P}(k)$ dominates, which follows the power law.

Let us consider a scale-free network in quantum system notation. The network's node is represented by a quantum state. The more nodes, the more superposed quantum states exist. Links between the quantum states can be represented through quantum links like the entanglement or generally through phase correlations. Remember that quantum links can be automatically propagated due to quantum swapping [10].

Till now we supposed all quantum states to be on the equal resolution level. We can move to quantum hierarchical systems (with architecture similar to the scale-free networks), where we must distinguish among various resolution levels as follows:

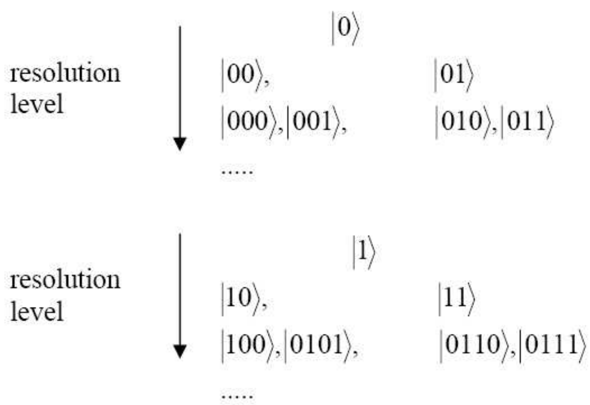

Fig. 1 *Hierarchy of quantum states on various resolution levels.*

4.1 Feedback hierarchical quantum systems

Now let us suppose the qubit $|\psi_{IN}\rangle = \alpha_0 \cdot |0\rangle + \alpha_1 \cdot |1\rangle$ is sent to the input of a very simple quantum feedback system given in Fig. 2. Let us suppose the quantum system S is defined by a quantum impulse response represented as the inner quantum state $|\psi_{INNER}\rangle = \beta_0 \cdot |0\rangle + \beta_1 \cdot |1\rangle$.

It is easy to track each feedback loop and identify the quantum output state:

$$
\begin{aligned}
&|\psi_{OUT}\rangle_1 = (\alpha_0 |0\rangle + \alpha_1 |1\rangle) \cdot (\beta_0 |0\rangle + \beta_1 |1\rangle)\\
&|\psi_{OUT}\rangle_2 = (\alpha_0 |0\rangle + \alpha_1 |1\rangle) \cdot (\beta_0 |0\rangle + \beta_1 |1\rangle)^2 + (\alpha_0 |0\rangle + \alpha_1 |1\rangle) \cdot (\beta_0 |0\rangle + \beta_1 |1\rangle)\\
&|\psi_{OUT}\rangle_3 = (\alpha_0 |0\rangle + \alpha_1 |1\rangle) \cdot (\beta_0 |0\rangle + \beta_1 |1\rangle)^3 + (\alpha_0 |0\rangle + \alpha_1 |1\rangle) \cdot (\beta_0 |0\rangle + \beta_1 |1\rangle)^2 +\\
&+ (\alpha_0 |0\rangle + \alpha_1 |1\rangle) \cdot (\beta_0 |0\rangle + \beta_1 |1\rangle)\\
&\ldots
\end{aligned}
\tag{37}
$$

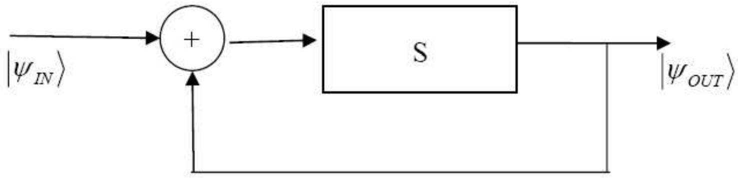

Fig. 2 *Quantum feedback system.*

In reality all superposed states on all resolution levels must be available in the final quantum output state:

$$|\psi_{OUT}\rangle = |\psi_{OUT}\rangle_1 + |\psi_{OUT}\rangle_2 + |\psi_{OUT}\rangle_3 + \dots \tag{38}$$

This task cannot be solved through the QTF because the condition (2) is not fulfilled.

In case the output of quantum system in Fig. 2 has the same dimension like its input, we can use a matrix representation (21). The quantum circuits can be solved with help of matrix algebra as (31) under the condition $B = 1$. In the case of QST application to the system in Fig. 2, the form (36) can be used under the condition $Q_2\left(\eta^{-1}\right) = 1$.

The generation of hierarchical systems by quantum feedback can yield into more complex circuits, where the links (entanglements) between states on different levels can be formed. Such networks can create interesting structures with many features. This instrument can be used as an approach to consciousness model that theoretically enables storage of infinity information in links. Unfortunately, deepest resolution levels can be accessible only with very low probability. There has to exist special filters that amplify such states at lower resolution areas.

4.2 Non-linear quantum systems

Similar hierarchical structures as those computed by feedback could be obtained by the *non-linear quantum system.* Let us suppose we have a quantum polynomial non-linear impulse function:

$$\begin{aligned}|\psi_{INNER}\rangle = \lambda_N \cdot (\beta_0 \cdot |0\rangle + \beta_1 \cdot |1\rangle)^N + \lambda_{N-1} \cdot (\beta_0 \cdot |0\rangle + \beta_1 \cdot |1\rangle)^{N-1} \dots \\ \dots + \lambda_1 \cdot (\beta_0 \cdot |0\rangle + \beta_1 \cdot |1\rangle),\end{aligned} \tag{39}$$

where $\lambda_i, i \in \{1, 2, .., N\}$ are complex parameters.

For input qubit $|\psi_{IN}\rangle = \alpha_0 \cdot |0\rangle + \alpha_1 \cdot |1\rangle$ sent into such a non-linear system, we can write the quantum output as:

$$\begin{aligned}&|\psi_{OUT}\rangle = \lambda_N \cdot (\beta_0 \cdot |0\rangle + \beta_1 \cdot |1\rangle)^N \cdot (\alpha_0 \cdot |0\rangle + \alpha_1 \cdot |1\rangle) + \dots \\ &\dots + \lambda_{N-1} \cdot (\beta_0 \cdot |0\rangle + \beta_1 \cdot |1\rangle)^{N-1} \cdot (\alpha_0 \cdot |0\rangle + \alpha_1 \cdot |1\rangle) + \dots \\ &\dots + \lambda_1 \cdot (\beta_0 \cdot |0\rangle + \beta_1 \cdot |1\rangle) \cdot (\alpha_0 \cdot |0\rangle + \alpha_1 \cdot |1\rangle)\end{aligned} \tag{40}$$

It is evident that the output (40) has similar behavior to the feedback system (38) which yields to scale-free structure of quantum network.

4.3 Complex hierarchical quantum networks

We can easily imagine parallel scale-free networks that interact among each other (for example through the entanglement or through quantum links – connections – among nodes of different networks).

Such complex and hierarchical networks (structures) have many possibilities how to store information on different resolution levels. A slight change of feedback or non-linearity can directly yield into amplification of concrete information on the selected resolution level.

5. Quantum Information Circuits

As expressed in [11], the information/knowledge circuit is defined through information flow Φ and information content I, analogically to current and voltage in the electrical engineering area. We can suppose that both Φ and I are prepared in superposed quantum states as stated in [12]. The quantum system is characterized by input and output parameters (2-port quantum system):

$$|\Phi_{IN}\rangle = \sum_{i=0}^{N_1} {}^{IN}\alpha_i \cdot \left|{}^{IN}\Phi_i\right\rangle, \ \ |I_{IN}\rangle = \sum_{j=0}^{M_1} {}^{IN}\beta_j \cdot \left|{}^{IN}I_j\right\rangle \tag{41}$$

$$|\Phi_{OUT}\rangle = \sum_{i=0}^{N_2} {}^{OUT}\alpha_i \cdot \left|{}^{OUT}\Phi_i\right\rangle, \ \ |I_{OUT}\rangle = \sum_{j=0}^{M_2} {}^{OUT}\beta_j \cdot \left|{}^{OUT}I_j\right\rangle. \tag{42}$$

The modeling of quantum information/knowledge circuits yields into connections of 2-port quantum subsystems defined above. The serial, parallel and feedback connections build more and more complex quantum information/knowledge systems with many features. The input and output quantum information power for subsystem (41), (42) was defined in [12].

As an example of possible interpretation of quantum information/knowledge circuits we can assume that the input information flow can be defined as a task per second, and the input information content as cost per task (cost can be sometimes better than energy, number of successful events, etc.). The output information flow can represent the received "know-how" and the content e.g. the earned money or "profit". The input parameters can represent the theoretical values (low cost, only intellectual effort) but the output values of quantum information/knowledge circuits can yield into very expensive changes of factory processes where a mistake can cost an enormous amount of money. This example can demonstrate that the quantum information/knowledge circuit can realize a controlled flow of energy (money) with respect to input information.

The first approach to the quantum information circuit yields into quantum automata that can generally realize q-bit functions as passing q-bits through quantum gates. The quantum gates can be ordered in very complex structures, including also feedbacks.

The (input) information flows (tasks) can be assigned to various subsystems together with the information content (cost per task). These two parameters can be mapped on a predefined hierarchical structure where each subsystem generates the output flow and content values (e.g. know-how, profit). All input and output values could be summarized and the final input/output values identified. This methodology can be used for assessment of various organizational structures or very complex hierarchical systems.

6. Conclusion

In this paper, we tried to analyze different representations of quantum subsystems with respect to their ordering into more complex circuits or networks. First of all, special features, such as additive principle of quantum states, were identified and an appropriate quantum representation, e.g. quantum transfer function, was introduced. Such simplification yields into easier analyzing of complex connected systems through serial, parallel or feedback links, as it is obvious in classical system theory.

The representation of complex quantum circuits or networks brought many new ideas as well as many questions. We were able to introduce hierarchical quantum systems there were similar to scale-free networks with many various resolution levels. The hierarchical system can be created through feedback or through quantum non-linear systems.

The non-linear quantum models similar to transistors seem to be a very promising area in electronics. It is possible that our quantum subsystems cover or can obtain the source of energy from its environment. Such energy can be used to amplify the output information power or to transfer e.g. the input information content into e.g. an output information flow. Such an approach is very close to autonomous living agents introduced by [13]. From this point of view, self-organization can be understood as controlled distribution of free energy. It is really what our quantum information/knowledge subsystem can do. It uses the input data (flow, content) to transfer free and non-organized energy to the output data (flow, content) where, at output, we work with much higher energies than at input. But what does it mean for non-organized energy? We can imagine the interface between our system and its environment. The controlled withdrawal of energy out of the system environment (order decreasing outside the system) yields in increasing the order of the studied system.

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