NEW RESULTS ON STABILITY OF STOCHASTIC NEURAL NETWORKS WITH MARKOVIAN SWITCHING AND MODE-DEPENDENT TIME-VARYING DELAYS*

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Abstract: This paper is concerned with the problem of exponential stability for a class of stochastic neural networks with Markovian switching and mode-dependent interval time-varying delays. A novel Lyapunov-Krasovskii functional is introduced with the idea of delay-partitioning, and a new exponential stability criterion is derived based on the new functional and free-weighting matrix method. This new criterion proves to be less conservative than the most existing results. Numerical examples are presented to illustrate the effectiveness of the proposed method.

Key words: *Lyapunov-Krasovskii functional, Markovian switching, mode-dependent time delays, neural networks, stochastic systems*

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1. Introduction

In the past years, neural networks (NNs) have been extensively studied due to the fact that they have found a large amount of successful applications in a variety of areas including pattern recognition, associative memory, and combinational optimization ([1], [2]). It is well known that these applications heavily depend on NNs' dynamic behaviors. Among the behaviors, stability is one of the most important ones that have received considerable research attention. In addition, time delays are frequently encountered in NNs, which are often the sources of instability and oscillations. Therefore, a great number of results on stability of delayed NNs have been reported in the literature (see, e.g., [3]-[7], and the references therein).

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As is well known, NNs could be stabilized or destabilized by certain stochastic inputs [8]. Many results have been reported on stability analysis for this class of NNs (see, e.g., [8]-[14], and the references therein). On the other hand, it has been shown in [15] that the switching between different NNs modes can be governed by a Markovian chain. NNs with Markovian switching are of great significance in modeling a class of NNs with finite network modes (see, e.g., [16]-[20], and the references therein). Recently, many researchers have been focused on the stability analysis for delayed stochastic NNs with Markovian switching, see [21]-[33].

It should be pointed that the time delays of aforementioned results are independent of the system modes. As is well known, mode-dependent time delays are of practical significance since the signal may switch between different modes and also propagate in a distributed way during a certain time period with the presence of an amount of parallel pathways [34]. Up to date, several results have been reported on stochastic Markovian jump neural networks with mode-dependent time-varying delays. In [35], a new Lyapunov-Krasovskii functional is chosen to handle Markovian jumping parameters, mixed mode-dependent time- varying delays, uncertainty, and Brownian motion. Delay-dependent conditions on mean square asymptotic stability are obtained in terms of LMIs. The problem of global exponential stability of neutral high-order stochastic Hopfield neural networks with Markovian switching and mixed mode-dependent time delays is investigated in [36]. In [37], the global exponential stabilization problem is investigated for a class of stochastic Cohen-Grossberg neural networks. A Lyapunov-Krasovskii functional that accounts for the mode-dependent mixed delays is introduced and stochastic analysis is conducted to derive stability criteria. However, these delay-dependent results assume the time delay is varying between zero and an upper bound, while in practice, the lower bound may not be restricted to be zero.

In this paper, we are concerned with the exponential stability conditions for stochastic neural networks with Markovian switching and mode-dependent interval time-varying delays. To this end, we introduce a new Lyapunov-Krasovskii functional based on the idea of partitioning the lower bound and then present a new delay-dependent stability criterion. Numerical examples are provided to illustrate the effectiveness and less conservatism of proposed method.

2. Problem formulation

Let $\{r_t, t \geq 0\}$ is a continuous-time Markov process with a right continuous trajectory taking values in a finite set $\mathbf{S} = \{1, 2, \ldots, s\}$ with transition probability matrix $\Lambda = \{\pi_{ij}\}$ given by

$$\mathbf{P}\left[r_{t+\Delta t} = j | r_t = i\right] = \begin{cases} \pi_{ij}\Delta + o\left(\Delta\right) & \text{if } j \neq i \\ 1 + \pi_{ij}\Delta + o\left(\Delta\right) & \text{if } j = i \end{cases}$$

where $\lim\limits_{\Delta \to 0} {}^{o\left(\Delta\right)}/_{\Delta} = 0$; $\pi_{ij} > 0,\ j \neq i$ and $\pi_{ii} = -\sum\limits_{j \neq i} \pi_{ij}$ for each $i \in \mathbf{S}$.

Fix a probability space $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$ and consider a class of stochastic neural networks with Markovian switching and mode-dependent time-varying delays as follows:

$$\begin{aligned} dx(t) = & \left[-A(r_t)x(t) + B(r_t)g\left(x(t)\right) + C(r_t)g\left(x\left(t-\tau(t,r_t)\right)\right) + J\right]dt \\ & + \left[D_1(r_t)x(t) + D_2(r_t)x\left(t-\tau(t,r_t)\right) + D_3(r_t)g\left(x(t)\right) + \right. \\ & \left. + D_4(r_t)g\left(x\left(t-\tau(t,r_t)\right)\right)\right]d\varpi(t) \end{aligned} \tag{1}$$

$$x(t) = \varphi(t), t \in [-\tau_2, 0] \tag{2}$$

where $x(t) \in \mathbf{R}^n$ is the state vector; $g\left(x(t)\right) = \left[g_1\left(x_1(t)\right), g_2\left(x_2(t)\right), \cdots, g_n\left(x_n(t)\right)\right] \in \mathbf{R}^n$ denotes the neuron activation function; $\varphi(t)$ is the initial condition; $\varpi(t)$ is a 1-D Brownian motion defined on probability space $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$; $A(r_t)$ is a positive diagonal matrix; $B(r_t)$, $C(r_t)$, $D_3(r_t)$ and $D_4(r_t)$ are the connection weight matrices with appropriate dimensions; $D_1(r_t) \in \mathbf{R}^{n\times n}$ and $D_2(r_t) \in \mathbf{R}^{n\times n}$ are known real constant matrices.

For notational simplicity, in the sequel, for each possible $r_t \in i$, $i \in \mathbf{S}$, a matrix $M(r_t)$ will be denoted by M_i; for example, $A\left(r_t\right)$ is denoted by A_i, $B(r_t)$ is denoted by B_i, and so on.

The mode-dependent time-varying delays satisfy the following condition:

$$0 < \tau_{1i} \le \tau_i(t) \le \tau_{2i} < \infty, \quad \dot{\tau}_i(t) \le \mu_i \tag{3}$$

where τ_{1i}, τ_{2i} and μ_i are constant for each $i \in \mathbf{S}$. Set $\tau_1 = \min\left\{\tau_i, i \in S\right\}$, $\tau_2 = \max\left\{\tau_i, i \in S\right\}$, and $\mu = \max\left\{\mu_i, i \in S\right\}$.

The following assumption is made on the neuron activation function.

Assumption 1 The neuron activation function$g(x)$is continuous and bounded and it satisfies the following condition:

$$0 \le \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \le l_j, \quad j = 1, 2, \ldots, n \tag{4}$$

for all $s_1, s_2 \in \mathbf{R}$, $s_1 \ne s_2$.

It should be pointed that Assumption 1 guarantees there is an equilibrium point for NN (1) by using Bouwer's fixed point problem. Let $x^* = [x_1^*, x_2^*, \ldots, x_n^*]$ be the equilibrium point. For the purpose of simplicity, we make the following transformation by the change of variables $y(t) = x(t) - x^*$. Under this transformation, NN (1) becomes:

$$\begin{aligned} dy(t) = & \left[-A(r_t)y(t) + B(r_t)f\left(y(t)\right) + C(r_t)f\left(y\left(t-\tau(t,r_t)\right)\right)\right]dt \\ & + \left[D_1(r_t)y(t) + D_2(r_t)y\left(t-\tau(t,r_t)\right) + D_3(r_t)f\left(y(t)\right) + \right. \\ & \left. + D_4(r_t)f\left(y\left(t-\tau(t,r_t)\right)\right)\right]d\varpi(t) \end{aligned} \tag{5}$$

where $f_j\left(y_j(t)\right) = g_j\left(x_j(t) + x_j^*\right) - g_j(x_j^*)$. It follows from (4) that the transformed neuron activation function $f(x)$ satisfy the following condition:

$$0 \le \frac{f_j(y_j)}{y_j} \le l_j, \quad f_j(0) = 0 \quad \forall y_j \ne 0, \quad j = 1, 2, \ldots, n \tag{6}$$

The following lemma will be used in the derivation of the main results.

Lemma 1 [38]

For any vectors $x, y \in \mathbf{R}^n$; matrices $P > 0$ with appropriate dimensions, then

$$2x^T y \leq x^T P^{-1} x + y^T P y \tag{7}$$

3. Main results

In this section, a new delay-dependent exponential stability criterion will be derived for NN (2).

Theorem 1 Given an integer $m \geq 1$, for time-varying delay $\tau_i(t)$ satisfying (3), the system (5) is globally mean-square exponentially stable , if there exist matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $Q_{3i} > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_1 > 0$, $R_3 > 0$, $R_4 > 0$, $X > 0$, $Y > 0$, $U = diag\{u_1, u_2, \ldots, u_n\} > 0$, $V = diag\{v_1, v_2, \ldots, v_n\} > 0$, such that the following LMIs hold for each $i \in \mathbf{S}$

$$\Omega_i = \begin{bmatrix} \Xi_{i1} & X & Y & \tau_1 X & \tau_{12} Y \\ * & -R_3 & 0 & 0 & 0 \\ * & * & -R_4 & 0 & 0 \\ * & * & * & -m\tau_1 R_1 & 0 \\ * & * & * & * & -m\tau_{12} R_2 \end{bmatrix} < 0 \tag{8}$$

$$\sum_{j=1}^{s} \pi_{ij} Q_{1j} \leq Q_1 \tag{9}$$

$$\sum_{j=1}^{s} \pi_{ij} Q_{2j} + \sum_{j=1, j\neq i}^{s} \pi_{ij} Q_{3j} \leq Q_2 \tag{10}$$

where

$$\tau_{12} = \tau_2 - \tau_1$$

$$\begin{aligned}
\Xi_{i1} \quad = \quad & W_1^T \left(\sum_{j=1}^{s} \pi_{ij} P_j + \tau_{12} Q_2 \right) W_1 + W_2^T \left(Q_{1i} + \frac{\tau_1}{m} Q_1 \right) W_2 - \\
& - W_3^T Q_{1i} W_3 - 2W_4^T U W_4 - 2W_5^T V W_5 + \\
& + W_6^T \left(Q_{2i} + Q_{3i} \right) W_6 - (1 - u_i) W_7^T Q_{2i} W_7 - W_8^T Q_{3i} W_8 + \\
& + sym \left(W_1^T P_i W_9 + X W_{10} + Y W_{11} + W_1^T U L W_4 + W_7^T V L W_5 \right) + \\
& + \frac{\tau_1}{m} W_9^T R_1 W_9 + \tau_{12} W_9^T R_2 W_9 + \\
& + \frac{\tau_1}{m} W_{12}^T R_3 W_{12} + \tau_{12} W_{12}^T R_4 W_{12} + W_{12}^T P_i W_{12},
\end{aligned}$$

$$W_1 = \begin{bmatrix} I_n & 0_{n,(m+4)n} \end{bmatrix}, \quad W_2 = \begin{bmatrix} I_{mn} & 0_{mn,5n} \end{bmatrix},$$

$$W_3 = \begin{bmatrix} 0_{mn,n} & I_{mn} & 0_{mn,4n} \end{bmatrix}, \quad W_4 = \begin{bmatrix} 0_{n,(m+3)n} & I_n & 0_{n,n} \end{bmatrix},$$

$$W_5 = \begin{bmatrix} 0_{n,(m+4)n} & I_n \end{bmatrix}, \quad W_6 = \begin{bmatrix} 0_{n,mn} & I_n & 0_{n,4n} \end{bmatrix}$$

$$W_7 = \begin{bmatrix} 0_{n,(m+1)n} & I_n & 0_{n,3n} \end{bmatrix}, \quad W_8 = \begin{bmatrix} 0_{n,(m+2)n} & I_n & 0_{n,2n} \end{bmatrix},$$

$$W_9 = \begin{bmatrix} -A_i & 0_{n,(m+2)n} & B_i & C_i \end{bmatrix}, \quad W_{10} = \begin{bmatrix} I_n & -I_n & 0_{n,(m+3)n} \end{bmatrix},$$

$$W_{11} = \begin{bmatrix} 0_{n,mn} & I_n & 0_{n,n} & -I_n & 0_{n,2n} \end{bmatrix},$$

$$W_{12} = \begin{bmatrix} D_{1i} & 0_{n,mn} & D_{2i} & 0_{n,n} & D_{3i} & D_{4i} \end{bmatrix}.$$

Proof For convenience, set

$$m(t) = -A(r_t)y(t) + B(r_t)f\left(y(t)\right) + C(r_t)f\left(y\left(t - \tau(t, r_t)\right)\right)$$

$$n(t) = D_1(r_t)y(t) + D_2(r_t)y\left(t - \tau(t, r_t)\right) + D_3(r_t)f\left(y(t)\right) + D_4(r_t)f\left(y\left(t - \tau(t, r_t)\right)\right)$$

Thus, system (5) can be represented as follows:

$$dy(t) = m(t)dt + n(t)d\varpi(t) \tag{11}$$

Define a new process $\{(y_t, r_t), t \geq 0\}$ by $y_t(s) = y(t+s)$, $-2\tau_2 \leq s \leq 0$, then $\{(y_t, r_t), t \geq 0\}$ is a Markov process with initial state $(\varphi(\cdot), r_0)$. Now take the stochastic Lyapunov–Krasovskii functional candidate as

$$\begin{aligned} V(y_t, r_t, t) \;=\; & y^T(t)P(r_t)y(t) + \int_{t-\frac{\tau_1}{m}}^{t} \Upsilon^T(\alpha)Q_1(r_t)\Upsilon(\alpha)d\alpha \\ & + \int_{t-\tau_i(t)}^{t-\tau_1} y^T(\alpha)Q_2(r_t)y(\alpha)d\alpha + \int_{t-\tau_2}^{t-\tau_1} y^T(\alpha)Q_3(r_t)y(\alpha)d\alpha \\ & + \int_{-\frac{\tau_1}{m}}^{0} \int_{t+\beta}^{t} m^T(\alpha)R_1 m(\alpha)d\alpha d\beta + \int_{-\tau_2}^{-\tau_1} \int_{t+\beta}^{t} m^T(\alpha)R_2 m(\alpha)d\alpha d\beta \\ & + \int_{-\frac{\tau_1}{m}}^{0} \int_{t+\beta}^{t} n^T(\alpha)R_3 n(\alpha)d\alpha d\beta + \int_{-\tau_2}^{-\tau_1} \int_{t+\beta}^{t} n^T(\alpha)R_4 n(\alpha)d\alpha d\beta \\ & + \int_{-\frac{\tau_1}{m}}^{0} \int_{t+\beta}^{t} \Upsilon^T(\alpha)Q_1\Upsilon(\alpha)d\alpha d\beta + \int_{-\tau_2}^{-\tau_1} \int_{t+\beta}^{t} y^T(\alpha)Q_2 y(\alpha)d\alpha d\beta \end{aligned} \tag{12}$$

where

$$\Upsilon(t) = \begin{bmatrix} y^T(t) & y^T(t - \frac{1}{m}\tau_1) & y^T(t - \frac{2}{m}\tau_1) & \cdots & y^T(t - \frac{m-1}{m}\tau_1) \end{bmatrix}^T.$$

By Itô's Lemma, we have

$$dV(y_t, r_t, t) = LV(y_t, r_t, t)dt + 2y^T(t)P(r_t)n(t)d\varpi(t),$$

where

$$\begin{aligned}
LV(y_t, r_t, t) &= 2y^T(t)P_i m(t) + y^T(t)\sum_{j=1}^{s}\pi_{ij}P_j y(t) + n^T(t)P_i n(t) \\
&+ \Upsilon^T(t)Q_{1i}\Upsilon(t) - \Upsilon^T(t-\tfrac{\tau_1}{m})Q_{1i}\Upsilon(t-\tfrac{\tau_1}{m}) \\
&+ \int_{t-\frac{\tau_1}{m}}^{t}\Upsilon^T(\alpha)\left(\sum_{j=1}^{s}\pi_{ij}Q_{1j}\right)\Upsilon(\alpha)d\alpha + y^T(t-\tau_1)Q_{2i}y(t-\tau_1) \\
&- (1-\mu_i)y^T\left(t-\tau_i(t)\right)Q_{2i}y\left(t-\tau_i(t)\right) + \sum_{j=1}^{s}\pi_{ij}\int_{t-\tau_j(t)}^{t-\tau_1}y^T(\alpha)Q_{2j}y(\alpha)d\alpha \\
&+ y^T(t-\tau_1)Q_{3i}y(t-\tau_1) - y^T(t-\tau_2)Q_{3i}y(t-\tau_2) \\
&+ \int_{t-\tau_2}^{t-\tau_1}y^T(\alpha)\left(\sum_{j=1}^{s}\pi_{ij}Q_{3j}\right)y(\alpha)d\alpha + \tfrac{\tau_1}{m}m^T(t)R_1 m(t) \\
&- \int_{t-\frac{\tau_1}{m}}^{t}m^T(\alpha)R_1 m(\alpha)d\alpha + \tau_{12}m^T(t)R_2 m(t) - \int_{t-\tau_2}^{t-\tau_1}m^T(\alpha)R_2 m(\alpha)d\alpha \\
&+ \tfrac{\tau_1}{m}n^T(t)R_3 n(t) - \int_{t-\frac{\tau_1}{m}}^{t}n^T(\alpha)R_3 n(\alpha)d\alpha + \tau_{12}n^T(t)R_4 n(t) \\
&- \int_{t-\tau_2}^{t-\tau_1}n^T(\alpha)R_4 n(\alpha)d\alpha + \tfrac{\tau_1}{m}\Upsilon^T(t)Q_1\Upsilon(t) - \int_{t-\frac{\tau_1}{m}}^{t}\Upsilon^T(\alpha)Q_1\Upsilon(\alpha)d\alpha \\
&+ \tau_{12}y^T(t)Q_2 y(t) - \int_{t-\tau_2}^{t-\tau_1}y^T(\alpha)Q_2 y(\alpha)d\alpha
\end{aligned} \tag{13}$$

From (11), for any appropriately dimensioned matrices X and Y, we have

$$2\zeta^T(t)X\left[y(t) - y(t-\frac{\tau_1}{m}) - \int_{t-\frac{\tau_1}{m}}^{t}m(\alpha)d\alpha - \int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha)\right] = 0$$

$$2\zeta^T(t)Y\left[y(t-\tau_1) - y(t-\tau_2) - \int_{t-\tau_2}^{t-\tau_1}m(\alpha)d\alpha - \int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha)\right] = 0 \tag{14}$$

where

$$\zeta(t) = \begin{bmatrix}\Upsilon^T(t) & y^T(t-\tau_1) & y^T\left(t-\tau_i(t)\right) & y^T(t-\tau_2) & f^T\left(y(t)\right) & f^T\left(y\left(t-\tau_i(t)\right)\right)\end{bmatrix}^T$$

From Lemma 1, we have

$$\begin{aligned}
-2\zeta^T(t)X\int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha) &\le \zeta^T(t)XR_3^{-2}X^T\zeta(t) + \\
&+ \left(\int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha)\right)^T R_3\left(\int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha)\right)
\end{aligned}$$

$$\begin{aligned}
-2\zeta^T(t)Y\int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha) &\le \zeta^T(t)YR_4^{-2}Y^T\zeta(t) + \\
&+ \left(\int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha)\right)^T R_4\left(\int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha)\right)
\end{aligned} \tag{15}$$

Noting (6), let $U = diag\{u_1, u_2, \ldots, u_n\}$ and $V = diag\{v_1, v_2, \ldots, v_n\}$. Then, we get

$$2\left[y^T(t)ULf\left(y(t)\right) - f^T\left(y(t)\right)Uf\left(y(t)\right)\right] \ge 0,$$

$$2\left[y^T\left(t-\tau_i(t)\right)VLf\left(y\left(t-\tau_i(t)\right)\right)-f^T\left(y\left(t-\tau_i(t)\right)\right)Vf\left(y\left(t-\tau_i(t)\right)\right)\right]\geq 0. \tag{16}$$

Then, it follows from (13-16) that

$$\begin{aligned}
LV(y_t,r_t,t) &= 2y^T(t)P_im(t)+y^T(t)\sum_{j=1}^{s}\pi_{ij}P_jy(t)+n^T(t)P_in(t)+\Upsilon^T(t)Q_{1i}\Upsilon(t)\\
&-\Upsilon^T(t-\tfrac{\tau_1}{m})Q_{1i}\Upsilon(t-\tfrac{\tau_1}{m})+\int_{t-\frac{\tau_1}{m}}^{t}\Upsilon^T(\alpha)\left(\sum_{j=1}^{s}\pi_{ij}Q_{1j}\right)\Upsilon(\alpha)d\alpha\\
&+y^T(t-\tau_1)Q_{2i}y(t-\tau_1)-(1-\mu_i)y^T\left(t-\tau_i(t)\right)Q_{2i}y\left(t-\tau_i(t)\right)\\
&+\sum_{j=1}^{s}\pi_{ij}\int_{t-\tau_j(t)}^{t-\tau_1}y^T(\alpha)Q_{2j}y(\alpha)d\alpha+y^T(t-\tau_1)Q_{3i}y(t-\tau_1)\\
&-y^T(t-\tau_2)Q_{3i}y(t-\tau_2)+\int_{t-\tau_2}^{t-\tau_1}y^T(\alpha)\left(\sum_{j=1}^{s}\pi_{ij}Q_{3j}\right)y(\alpha)d\alpha\\
&+\tfrac{\tau_1}{m}m^T(t)R_1m(t)-\int_{t-\frac{\tau_1}{m}}^{t}m^T(\alpha)R_1m(\alpha)d\alpha+\tau_{12}m^T(t)R_2m(t)\\
&-\int_{t-\tau_2}^{t-\tau_1}m^T(\alpha)R_2m(\alpha)d\alpha+\tfrac{\tau_1}{m}n^T(t)R_3n(t)-\int_{t-\frac{\tau_1}{m}}^{t}n^T(\alpha)R_3n(\alpha)d\alpha\\
&+\tau_{12}n^T(t)R_4n(t)-\int_{t-\tau_2}^{t-\tau_1}n^T(\alpha)R_4n(\alpha)d\alpha+\tfrac{\tau_1}{m}\Upsilon^T(t)Q_1\Upsilon(t)\\
&-\int_{t-\frac{\tau_1}{m}}^{t}\Upsilon^T(\alpha)Q_1\Upsilon(\alpha)d\alpha+\tau_{12}y^T(t)Q_2y(t)-\int_{t-\tau_2}^{t-\tau_1}y^T(\alpha)Q_2y(\alpha)d\alpha\\
&+2\zeta^T(t)X\left[y(t)-y(t-\tfrac{\tau_1}{m})\right]-2\zeta^T(t)X\int_{t-\frac{\tau_1}{m}}^{t}m(\alpha)d\alpha+\zeta^T(t)XR_3^{-1}X^T\zeta(t)\\
&+\left(\int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha)\right)^TR_3\left(\int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha)\right)+2\zeta^T(t)Y\left[y(t-\tau_1)-y(t-\tau_2)\right]\\
&-2\zeta^T(t)Y\int_{t-\tau_2}^{t-\tau_1}m(\alpha)d\alpha+\zeta^T(t)YR_4^{-1}Y^T\zeta(t)\\
&+\left(\int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha)\right)^TR_4\left(\int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha)\right)+2y^T(t)ULf\left(y(t)\right)\\
&-2f^T\left(y(t)\right)Uf\left(y(t)\right)+2y^T\left(t-\tau_i(t)\right)VLf\left(y\left(t-\tau_i(t)\right)\right)\\
&-2f^T\left(y\left(t-\tau_i(t)\right)\right)Vf\left(y\left(t-\tau_i(t)\right)\right)+\tfrac{\tau_1}{m}\zeta^T(t)XR_1^{-1}X^T\zeta(t)\\
&-\int_{t-\frac{\tau_1}{m}}^{t}\zeta^T(t)XR_1^{-1}X^T\zeta(t)d\alpha\\
&+\tau_{12}\zeta^T(t)YR_2^{-1}Y^T\zeta(t)-\int_{t-\tau_1}^{t-\tau_1}\zeta^T(t)YR_2^{-1}Y^T\zeta(t)d\alpha\\
&\leq\Theta_i-\int_{t-\frac{\tau_1}{m}}^{t}n^T(\alpha)R_3n(\alpha)d\alpha-\int_{t-\tau_2}^{t-\tau_1}n^T(\alpha)R_4n(\alpha)d\alpha\\
&-\int_{t-\frac{\tau_1}{m}}^{t}\left[\zeta^T(t)X+m^T(\alpha)R_1\right]R_1^{-1}\left[X^T\zeta(t)+R_1m(\alpha)\right]d\alpha\\
&-\int_{t-\tau_2}^{t-\tau_1}\left[\zeta^T(t)Y+m^T(\alpha)R_2\right]R_2^{-1}\left[Y^T\zeta(t)+R_2m(\alpha)\right]d\alpha\\
&+\left(\int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha)\right)^TR_3\left(\int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha)\right)\\
&+\left(\int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha)\right)^TR_4\left(\int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha)\right)\\
&+\int_{t-\frac{\tau_1}{m}}^{t}\Upsilon^T(\alpha)\left(\sum_{j=1}^{s}\pi_{ij}Q_{1j}\right)\Upsilon(\alpha)d\alpha-\int_{t-\frac{\tau_1}{m}}^{t}\Upsilon^T(\alpha)Q_1\Upsilon(\alpha)d\alpha\\
&+\sum_{j=1}^{s}\pi_{ij}\int_{t-\tau_j(t)}^{t-\tau_1}y^T(\alpha)Q_{2j}y(\alpha)d\alpha+\int_{t-\tau_2}^{t-\tau_1}y^T(\alpha)\left(\sum_{j=1}^{s}\pi_{ij}Q_{3j}\right)y(\alpha)d\alpha\\
&-\int_{t-\tau_2}^{t-\tau_1}y^T(\alpha)Q_2y(\alpha)d\alpha
\end{aligned} \tag{17}$$

where

$$
\begin{aligned}
\Theta_i \quad = \quad & 2y^T(t)P_i m(t) + y^T(t)\sum_{j=1}^{s}\pi_{ij}P_j y(t) + n^T(t)P_i n(t) + \Upsilon^T(t)Q_{1i}\Upsilon(t) \\
& -\Upsilon^T(t-\frac{\tau_1}{m})Q_{1i}\Upsilon(t-\frac{\tau_1}{m}) + y^T(t-\tau_1)Q_{2i}y(t-\tau_1) \\
& -(1-\mu_i)y^T\left(t-\tau_i(t)\right)Q_{2i}y\left(t-\tau_i(t)\right) + y^T(t-\tau_1)Q_{3i}y(t-\tau_1) \\
& -y^T(t-\tau_2)Q_{3i}y(t-\tau_2) + \frac{\tau_1}{m}m^T(t)R_1 m(t) + \tau_{12}m^T(t)R_2 m(t) \\
& +\frac{\tau_1}{m}n^T(t)R_3 n(t) + \tau_{12}n^T(t)R_4 n(t) + \frac{\tau_1}{m}\Upsilon^T(t)Q_1\Upsilon(t) + \tau_{12}y^T(t)Q_2 y(t) \\
& +2\zeta^T(t)X\left[y(t)-y(t-\frac{\tau_1}{m})\right] + \zeta^T(t)XR_3^{-1}X^T\zeta(t) \\
& +2\zeta^T(t)Y\left[y(t-\tau_1)-y(t-\tau_2)\right] + \zeta^T(t)YR_4^{-1}Y^T\zeta(t) + 2y^T(t)ULf\left(y(t)\right) \\
& -2f^T\left(y(t)\right)Uf\left(y(t)\right) + 2y^T\left(t-\tau_i(t)\right)VLf\left(y\left(t-\tau_i(t)\right)\right) \\
& -2f^T\left(y\left(t-\tau_i(t)\right)\right)Vf\left(y\left(t-\tau_i(t)\right)\right) + \frac{\tau_1}{m}\zeta^T(t)XR_1^{-1}X^T\zeta(t) \\
& +\tau_{12}\zeta^T(t)YR_2^{-1}Y^T\zeta(t) \\
= \quad & \zeta^T(t)\left[\Xi_{i1}+\Xi_{i2}\right]\zeta(t)
\end{aligned}
$$

$$\Xi_{i2} = XR_3^{-1}X^T + YR_4^{-1}Y^T + \frac{\tau_1}{m}XR_1^{-1}X^T + \tau_{12}YR_2^{-1}Y^T.$$

Noting $\pi_{ij} > 0$ for$j \neq i$ and $\pi_{ii} < 0$, then we have

$$\sum_{j=1}^{s}\pi_{ij}\int_{t-\tau_j(t)}^{t-\tau_1}x^T(\alpha)Q_{2j}x(\alpha)d\alpha \leq \int_{t-\tau_2}^{t-\tau_1}x^T(\alpha)\left(\sum_{j=1,j\neq i}^{s}\pi_{ij}Q_{2j}\right)x(\alpha)d\alpha \tag{18}$$

Since

$$
\begin{aligned}
&\mathbf{E}\left\{\int_{t-\frac{\tau_1}{m}}^{t}n^T(\alpha)R_3(\alpha)d\alpha\right\} = \mathbf{E}\left\{\left[\int_{t-\frac{\tau_1}{m}}^{t}n^T(\alpha)d\varpi(\alpha)\right]R_3\left[\int_{t-\frac{\tau_1}{m}}^{t}n(\alpha)d\varpi(\alpha)\right]\right\} \\
&\mathbf{E}\left\{\int_{t-\tau_2}^{t-\tau_1}n^T(\alpha)R_4(\alpha)d\alpha\right\} = \mathbf{E}\left\{\left[\int_{t-\tau_2}^{t-\tau_1}n^T(\alpha)d\varpi(\alpha)\right]R_4\left[\int_{t-\tau_2}^{t-\tau_1}n(\alpha)d\varpi(\alpha)\right]\right\}
\end{aligned}
\tag{19}
$$

Based on (9), (10), (17), (18), (19), we have

$$\mathbf{E}LV(y_t, r_t, t) \leq \mathbf{E}\left\{\zeta^T(t)\left[\Xi_{i1}+\Xi_{i2}\right]\zeta(t)\right\} \tag{20}$$

From (8) and using Schur complement lemma, it is easy to see that there exists a scalar $\varepsilon = \lambda_{\min}\left(-\Omega_i\right) > 0$ such that

$$\mathbf{E}LV(y_t, r_t, t) \leq -\varepsilon\mathbf{E}\left\{\left|y(t)\right|^2\right\} \tag{21}$$

Following the similar method of [26], we can prove the system (5) is mean-square exponential stability. This completes the proof.

Remark 1 Based on the idea of partitioning the lower bound, a new stability criterion for stochastic NNs with mode-dependent interval time-varying delays and Markovian switching is proposed. The criterion is formulated in terms of LMI, which can be easily solved by standard software. The reduced conservatism benefits from the construction of the new Lyapunov-Krasovskii functional. The parameter m refers to the number of delay partitioning, and it has relation to the conservatism. It is worth mentioning that the conservatism reduction increases as the delay fractioning becomes thinner.

Remark 2 From the derivation of Theorem 1, we can see that the matricesQ_{1i}, Q_{2i}, Q_{3i} are selected to be mode dependent and the free matrices X and Y are used, which can deduce some conservatism in some sense. In addition, for simplicity only, we do not consider uncertainties in our models. The proposed method can also be easily extended to systems with multiple and distributed delays.

For convenience in comparison, we set $\tau_{1i} = 0$, $i \in \mathbf{S}$. Then the time delay $\tau_i(t)$ satisfies

$$0 < \tau_i(t) \le \tau_{2i} < \infty, \quad \dot{\tau}_i(t) \le \mu_i \tag{22}$$

By the way of partitioning the upper bound τ_2, we have the following corollary.

Corollary 1 Given an integer $m \ge 1$, for time-varying delay $\tau_i(t)$ satisfying (22), the system (5) is globally mean-square exponentially stable , if there exist matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$,$X > 0$, $U = diag\{u_1, u_2, \ldots, u_n\} > 0$, $V = diag\{v_1, v_2, \ldots, v_n\} > 0$, such that the following LMIs hold for each $i \in \mathbf{S}$

$$\Omega_i = \begin{bmatrix} \Xi_{i1} & X & \tau_2 X \\ * & -R_2 & 0 \\ * & * & -m\tau_2 R_1 \end{bmatrix} < 0 \tag{23}$$

$$\sum_{j=1}^{s} \pi_{ij} Q_{1j} \le Q_1 \tag{24}$$

$$\sum_{j=1, j\ne i}^{s} \pi_{ij} Q_{2j} \le Q_2 \tag{25}$$

where

$$\begin{aligned}
\Xi_{i1} = {} & W_1^T \left(\sum_{j=1}^{s} \pi_{ij} P_j + Q_{2i} + \tau_2 Q_2 \right) W_1 + W_2^T \left(Q_{1i} + \tfrac{\tau_2}{m} Q_1 \right) W_2 - W_3^T Q_{1i} W_3 \\
& - 2W_4^T U W_4 - 2W_5^T V W_5 \\
& - (1-u_i) W_6^T Q_{2i} W_6 + sym\left(W_1^T P_i W_7 + X W_8 + W_1^T U L W_4 + W_6^T V L W_5 \right) \\
& + \tfrac{\tau_2}{m} W_7^T R_1 W_7 + \tfrac{\tau_2}{m} W_9^T R_2 W_9 + W_9^T P_i W_9
\end{aligned}$$

$$W_1 = \begin{bmatrix} I_n & 0_{n,(m+3)n} \end{bmatrix}, \quad W_2 = \begin{bmatrix} I_{mn} & 0_{mn,4n} \end{bmatrix},$$

$$W_3 = \begin{bmatrix} 0_{mn,n} & I_{mn} & 0_{mn,3n} \end{bmatrix}, \quad W_4 = \begin{bmatrix} 0_{n,(m+2)n} & I_n & 0_{n,n} \end{bmatrix},$$

$$W_5 = \begin{bmatrix} 0_{n,(m+3)n} & I_n \end{bmatrix}, \quad W_6 = \begin{bmatrix} 0_{n,(m+1)n} & I_n & 0_{n,2n} \end{bmatrix},$$

$$W_7 = \begin{bmatrix} -A_i & 0_{n,(m+1)n} & B_i & C_i \end{bmatrix}, \quad W_8 = \begin{bmatrix} I_n & -I_n & 0_{n,(m+2)n} \end{bmatrix},$$
$$W_9 = \begin{bmatrix} D_{1i} & 0_{n,mn} & D_{2i} & D_{3i} & D_{4i} \end{bmatrix}.$$

Proof Choose a Lyapunov-Krasovskii functional candidate as follows:

$$\begin{aligned} V(y_t, r_t, t) &= y^T(t)P(r_t)y(t) + \int_{t-\frac{\tau_2}{m}}^{t} \hat{\Upsilon}^T(\alpha)Q_1(r_t)\hat{\Upsilon}(\alpha)d\alpha \\ &+ \int_{t-\tau_i(t)}^{t} y^T(\alpha)Q_2(r_t)y(\alpha)d\alpha + \int_{-\frac{\tau_2}{m}}^{0} \int_{t+\beta}^{t} m^T(\alpha)R_1m(\alpha)d\alpha d\beta \\ &+ \int_{-\frac{\tau_2}{m}}^{0} \int_{t+\beta}^{t} n^T(\alpha)R_2n(\alpha)d\alpha d\beta + \int_{-\frac{\tau_2}{m}}^{0} \int_{t+\beta}^{t} \hat{\Upsilon}^T(\alpha)Q_1\hat{\Upsilon}(\alpha)d\alpha d\beta \\ &+ \int_{-\tau_2}^{0} \int_{t+\beta}^{t} y^T(\alpha)Q_2y(\alpha)d\alpha d\beta \end{aligned} \tag{26}$$

where

$$\hat{\Upsilon}(t) = \begin{bmatrix} y^T(t) & y^T(t - \frac{1}{m}\tau_2) & y^T(t - \frac{2}{m}\tau_2) & \cdots & y^T(t - \frac{m-1}{m}\tau_2) \end{bmatrix}^T.$$

With the same method presented in proving Theorem 1, we can get the result in Corollary 1.

Remark 3 It should be pointed out that even with the case $m = 1$, our method still outperforms the recent results [30], [35], which will be illustrated via examples in the next section. The reduced conservatism of Corollary 1 benefits from the matricesQ_{1i},Q_{2i}, which are selected to be mode-dependent in our paper.

4. Numerical examples

Example 1 Consider a stochastic neural network with Markovian switching and time-varying delay with two modes, that is, $S = \{1, 2\}$. Associated with modes 1 and 2, the system matrices are given by

$$A_1 = \begin{bmatrix} 2.0 & 0.0 \\ 0.0 & 3.0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 2.0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.4 & -0.1 \\ 0.5 & 0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 & -0.3 \\ 0.4 & 0.6 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.2 & 0.4 \\ 0.1 & -0.3 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -1.5 & 0.9 \\ 1.1 & -0.8 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 0.2 & 0.4 \\ 0.5 & 0.2 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} -0.1 & 0.3 \\ 0.3 & 0.5 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} -0.2 & 0.4 \\ 0.3 & 0.5 \end{bmatrix},$$

$$D_{31} = \begin{bmatrix} 0.2 & 0.4 \\ -0.8 & 0.1 \end{bmatrix}, \quad D_{32} = \begin{bmatrix} 0.1 & 0.2 \\ -0.4 & 0.3 \end{bmatrix},$$

$$D_{41} = \begin{bmatrix} 0.1 & 0.5 \\ 1.1 & 0.3 \end{bmatrix}, \quad D_{42} = \begin{bmatrix} 0.3 & 0.2 \\ 1.1 & 0.7 \end{bmatrix}.$$

The switching matrix

$$\Pi = \begin{bmatrix} -0.8 & 0.8 \\ 0.9 & -0.9 \end{bmatrix}.$$

We assume $\mu_1 = \mu_1 = \mu$, the comparison results of maximum time delay τ_2 via different μ are given in Tab. I. It is clearly seen that our results outperform those in [30], [35] even with the case $m = 1$. For example, when $\mu = 0.2$, the maximum upper delay bounds in [30] and [35] are 0.4298 and 0.4502 respectively, while the result in Corollary 1 is 1.8945 with the case $m = 1$, which is bigger than the result in [30] and [35]. The reduced conservatism of Corollary 1 benefits from the matrices Q_{1i},Q_{2i} in (26), which are selected to be mode-dependent in our paper.In addition, from Tab. I, it is easy to see that when the delay partitioning number m becomes larger, the conservatism of the results is further reduced.

u	0.2	0.5	0.8
[30]	0.4298	0.1849	infeasible
[35]	0.4502	0.2718	infeasible
Corollary 1, m=1	1.8945	0.9262	0.6821
Corollary 1, m=2	2.3842	1.3817	0.9153
Corollary 1, m=3	2.4873	1.4873	1.0372
Corollary 1, m=4	2.5017	1.5103	1.0826

Tab. I *Maximum delay bound τ_2 via different methods.*

Example 2 Consider a stochastic neural network with Markovian switching and mode-dependent interval time-varying delay with two modes and the following parameters:

$$A_1 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 2.0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.7 & 0.4 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.4 & 0.7 \\ 0.3 & -0.5 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.9 & 0.7 \\ 0.4 & -1.8 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 0.1 & 0.4 \\ 0.6 & 0.2 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & 0.1 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0.1 & 0.3 \\ -0.2 & 0.4 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0.2 & 0.6 \\ -0.5 & 0.5 \end{bmatrix},$$

$$D_{31} = \begin{bmatrix} 0.2 & 0.4 \\ -0.2 & 0.6 \end{bmatrix}, \quad D_{32} = \begin{bmatrix} 0.3 & 0.2 \\ -0.1 & 0.3 \end{bmatrix},$$

$$D_{41} = \begin{bmatrix} 0.4 & 0.7 \\ 1.2 & 0.5 \end{bmatrix}, \quad D_{42} = \begin{bmatrix} 0.2 & 0.2 \\ 1.3 & -0.7 \end{bmatrix},$$

The switching matrix

$$\Pi = \begin{bmatrix} -1.8 & 1.8 \\ 1.5 & -1.5 \end{bmatrix}.$$

For given $u_1 = 0.4$, $u_2 = 0.5$, the maximum τ_2 via different lower bound τ_1 are given in Tab. II. From Tab. II, it is clear to see that when the delay partitioning number m becomes larger, the conservatism of results is further reduced. For example, with the case $\tau_1 = 0.5$, the maximum upper delay bounds are 2.7491 ($m = 1$), 3.2893 ($m = 2$), 3.3718 ($m = 3$), and 3.4162 ($m = 4$) respectively. It should be pointed out that when the delay partitioning number m becomes larger, the computational cost increases. This is reasonable since m is related to the decision variables. Thus, the larger m indicates that the solution can be searched in a wider space and a longer maximum allowable delay bound τ_2 can be obtained.

τ_1	0.1	0.5	0.8
Theorem 1, m=1	1.9372	2.7491	3.8761
Theorem 1, m=2	2.1740	3.2893	4.1762
Theorem 1, m=3	2.2103	3.3718	4.3291
Theorem 1, m=4	2.2971	3.4162	4.3370

Tab. II *Maximum delay bound τ_2 via different τ_1.*

5. Conclusions

In this paper, the problem of exponential stability for stochastic neural networks with Markovian switching and mode-dependent interval time-varying delays has been investigated. With the idea of time delay partitioning, a new Lyapunov-Krasovskii functional has been proposed. Based on the new functional and free-weighting matrix method, a less conservative stability criterion has been derived. Numerical examples have been given to show the effectiveness and advantages of proposed method.

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