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# DYNAMICAL PROPERTIES OF FIRING PATTERNS IN THE HUBER-BRAUN COLD RECEPTOR MODEL IN RESPONSE TO EXTERNAL CURRENT STIMULI

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**Abstract:** We have studied the role of external current stimuli in a four-dimensional Hodgkin-Huxley-type model of cold receptor in this paper. Firstly, we researched its firing patterns from direct current (DC) and alternating current (AC) stimuli. Under different values of DC stimulus intensity, interspike intervals (ISIs) with period-doubling bifurcation phenomena appeared. Second, research has shown that neurons are extremely sensitive to changes in the frequency and amplitude of the current used to stimulate them. As the stimulus frequency increased, discharge rhythms emerged ranging from burst firing to chaotic firing and spiking firing. Meanwhile, various phase-locking patterns have been studied in this paper, such as  $p : 1$  ( $p > 1$ ),  $1 : q$  ( $q > 1$ ),  $2 : q$  ( $q > 1$ ) and  $p : q$  ( $p, q > 1$ ), etc. Finally, based on the fast-slow dynamics analysis, codimension-two bifurcation analysis of the fast subsystem was performed in the parameter  $(a_{sr}, B)$ -plane. We mainly investigated cusp bifurcation, fold-Hopf bifurcation, Bogdanov-Takens bifurcation and generalized Hopf bifurcation. These results revealed the effect of external current stimuli on the neuronal discharge rhythm and were instructive for further understanding the dynamical properties and mechanisms of the Huber-Braun model.

Key words: *firing patterns, phase-locking, interspike intervals (ISIs), firing rate, fast-slow dynamics analysis, Huber-Braun model*

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## 1. Introduction

External current stimuli can change the dynamics of nonlinear systems qualitatively. And a lot of experiment data indicates that a neuron has a tendency to burst depending on many internal and external factors, such as all kinds of ion channels, ion concentration, depolarizing currents and membrane capacitance, etc.

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