TOWARDS COMPLEX SYSTEM THEORY

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tutorial

Abstract: This tutorial summarizes the new approach to complex system theory that comes basically from physical – information analogies. The information components and gates are defined in a similar way as components in electrical or mechanical engineering. Such approach enables the creation of complex networks through their serial, parallel or feedback ordering. Taking into account wave probabilistic functions in analogy with quantum physics, we can enrich the system theory with features such as entanglement. It is shown that such approach can explain emergencies and self-organization properties of complex systems.

Key words: *Complex system theory, knowledge, quantum information systems, information power, information physics, self-organization, smart systems*

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1. Introduction

Information was interestingly described by George Bernard Shaw: "If you have an apple and I have an apple, and we exchange apples, we both still only have one apple. But if you have an idea (a piece of information) and I have an idea, and exchange ideas (this information), we each now have two ideas (two pieces of information)." Such example supposes our memory is a basis for system's specification: the *system* maps inputs into state values, and inputs and states into system's outputs.

Understanding the complex system is as if we were building a house. We need material (or mass), as well as plenty of workers (or energy), but without the knowledge of the plans as for when and how to build, we cannot erect the house. Information and knowledge are therefore the things that enrich the complex system theory and afterwards also natural sciences, enabling them to describe more faithfully the world around us.

The concept of data means a change of state, for example from 0 to 1 or from 1 to 0, where the state vector is not necessarily only digital or one-dimensional. Every such change can be described with the use of a quantity of *information* in bits.

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Information theory was founded by Claude Shannon [12] and his colleagues in the 1940s and was associated with coding and data transmission, especially in the newly emerging field of radar systems, which became a component of defensive systems during the Second World War.

Syntactic (Shannon) information was defined as the degree of probability of a given event and answered the question how often a message appears. For example, by telling you that the solar system will cease to exist tomorrow, I will be giving you the maximum information possible, because the probability of this phenomenon occurring is nearly equal to zero. The probability model of information defined in this way has been used for the design of self-repairing codes, digital modulations and other technical applications. Telecommunications specialists and radio engineers were concentrating on a probabilistic description of the encoded data and minimizing of probability errors during the data transmission.

There is a lot of approaches of how to extract information or eliminate entropy. *Bayes method* [11], which interprets the density of probability not as a description of a random quantity, but rather as a description of the uncertainty of the system, i.e. how much information is available about the monitored system. The system itself might be completely deterministic (describable without probability theory), but there may be very little information about the system available. When performing continuous measurement, we obtain more and more data, and therefore more information about our system, and thus our system begins to appear to us more definite. The elimination of uncertainty therefore increases the quantity of information we have about the monitored system.

Once uncertainty has been eliminated, one may proceed to the interpretation of information, or in other words, to the determination of how to reconstruct the described system, or how to build a more or less perfect model of it using the information [38]. This task already belongs to the theory of systems, where it is necessary to identify the status parameters, individual processes of the system etc. As a result, a knowledge system emerges, which is able to describe the given object appropriately.

The model-theoretical work of *semantic information* was done by Carnap and Bar-Hiller [1]. On the other hand, semantic information asks how often a message is true. Zadeh [41] introduced the theory of *fuzzy sets*, specific tool that maps a value for which an element is or is not a member of a set, expressed as a number between zero and one.

Models of complex systems are based on knowledge from *information science* that has been gathered over the years in classical physics, a specialized part of which is called *information physics* [13]. At present, this discipline is still in its infancy, but many discoveries have already been made, e.g. by Vedral [37] and some scientists have realized that without basic theories in this area, the further development of *complex system theory* will not be possible.

Section 2 presents author's approach to models of complex information systems based on physical-information analogies together with additional consequences of such way of thinking, e.g. information elements, that enables resonances or information networks with feedbacks, etc. Section 3 extends this approach to wave probabilistic functions that are more appropriate for models of soft systems. Quantum models can benefit from special features of wave probabilities like, e.g. entanglement, entanglement swapping or quantisation. Some examples of possible applications are given in Section 4. Section 5 concludes the paper.

2. Models of complex information systems

2.1 Information – physical analogies

In [22] the extended Frege's concept of information the image was presented based on results [9, 27, 39]. In Fig. 1 basic information quantities are given:

$O_i(t)$ – a set of rated quantities on an object,
$P_i(t)$ – a set of states,
$\Phi_i(t)$ – a set of syntactic strings (data flow),
$I_i(t)$ – a set of information images of state quantities.

In physics, the state is a complete description of a system in terms of parameters at a particular moment in time. Thermodynamic state is a set of physical quantities (e.g. temperature, pressure, and composition) describing variable properties.

Other parameters representing links between information quantities:

a_{OP} – identification,
a_{PO} – invasivity,
$a_{P\Phi}$ – projection in a set of symbols and syntactical strings,
$a_{\Phi P}$ – uncertainty correction and identification,
$a_{\Phi I}$ – interpretation, information origin,
$a_{I\Phi}$ – language constructs reflection,
a_{IO} – relation of functions and structural regularity,
a_{OI} – integrity verification.

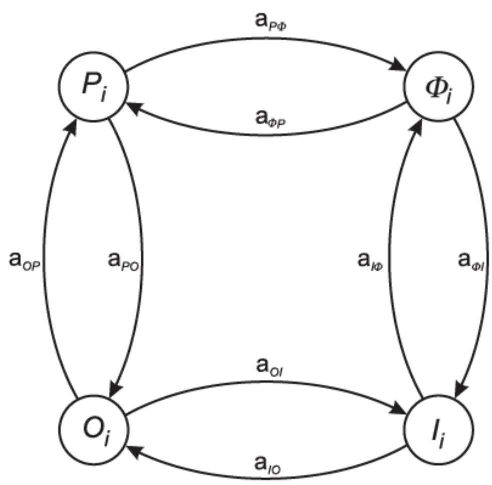


Fig. 1 *Frege's functional concept of information image origin and action.*

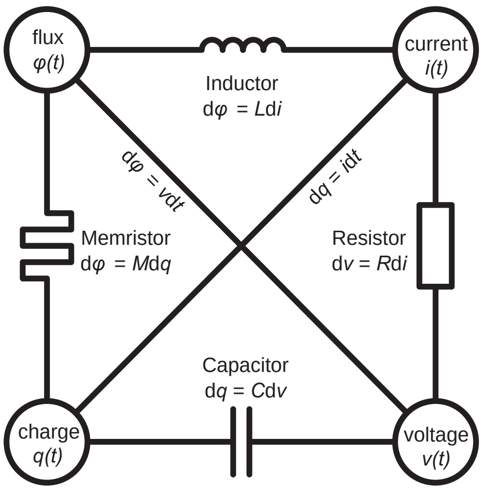


Fig. 2 *Chua's concept of electrical quantities.*

The circuit theoretician Chua [2] introduced the basic concept of electrical components together with the relations between them as it is shown in Fig. 2. There are six different mathematical relations connecting pairs of the four fundamental circuit variables:

$q(t)$	–	charge,
$\varphi(t)$	–	magnetic flux,
$i(t)$	–	electric current,
$v(t)$	–	voltage.

From electrical variables definition we know that the charge is the time integral of the current. Faraday's law tells us that the flux is the time integral of the electromotive force, or voltage. There should be four basic circuit elements described by relations between variables: resistor, inductor, capacitor and memristor. Chua's concept is famous due to an envisioned new electrical component named "memristor" that provides a functional relation between charge $q(t)$ and flux $\varphi(t)$.

Let us use the analogies between Chua's electrical and Frege's information concepts. Electrical charge $q(t)$ is measured in coulombs [c] and can be understood as basic electrical unit in analogy with *the information unit* measured in bits [bit]. Quantum physics really defines the quantum information unit called q-bit. We speak of (input/output) data changes $O_i(t)$ through which the system's model can be estimated.

Magnetic flux $\varphi(t)$ is naturally related to *the system states* $P_i(t)$ which should be viewed as the piece of extracted knowledge (e.g. parameters estimated based on the observed data, understanding why system behaves in such a way, etc.). Magnetic flux is measured in webers [wb] corresponding to Joule multiplied by second and divided by coulomb [J.s/c]. In agreement with our analogy the states $P_i(t)$ are measured in [J.s/bit] which notes how much energy can be extracted/delivered based on one bit of information within one second.

Electrical current $i(t)$ measured in coulomb per second [c/s] is related to *the information (syntax) flow* $\Phi_i(t)$ that describes dynamical property of data changes in bits per second [bit/s]. The information flow $\Phi_i(t)$ typically represents the system input/output data flows per time.

Electrical voltage $v(t)$ leads to the analogical definition of *the information (semantic) content* $I_i(t)$ which characterizes the knowledge content measured in Joule per bit [J/bit]. For information systems (IT/ICT), the information content in [J/bit] can be alternatively defined as the number of "success events" caused by the receipt of one bit of information [39].

2.2 Information elements

The data $O_i(t)$ carries a piece of knowledge available in states $P_i(t)$ or vice versa, the piece of knowledge $P_i(t)$ can be represented by the data set $O_i(t)$. Linear relation tells us that the more data sets $O_i(t)$ are available the more knowledge can be extracted. Non-linear relation yields to conclusion that data overwhelming need not bring us additional knowledge. Such attribute can be modelled by *information memristor* – a component that possesses a knowledge memory (e.g. a priori information from historical data).

It is evident that *information resistor* gives us relation between the information flow $\Phi_i(t)$ and the information content $I_i(t)$. *Information capacitor* explains links between the measured data changes $O_i(t)$ and the information content $I_i(t)$. The bigger data set is observed the better system description is available (analogy with capacitor charging). Information capacitor can be understood as a knowledge storage component.

Information inductor describes relation between the information flow $\Phi_i(t)$ and the knowledge piece available in the states $P_i(t)$. The more knowledge in states $P_i(t)$ the more significant the information flow $\Phi_i(t)$. Similarly we can speak about an information inductor as the storage component of the information flow (analogy with coil property).

2.3 Information resonance

We can continue in our way of thinking and define basic principles of *information resonance*. For example, we can imagine two hemispheres of our brain. The left hemisphere plays the role of an information inductor – the source of information flow $\Phi_i(t)$ based on identified knowledge. To the contrary, the right hemisphere could be described as an information capacitor – the source of information content $I_i(t)$ based on the observed data – analytical part that try to interpret the available data. The resonance principle can be modeled by the means of co-operation between both hemispheres. The bigger data flow $\Phi_i(t)$ is generated by the left hemisphere the higher knowledge $I_i(t)$ can be extracted by right hemisphere. Higher knowledge $I_i(t)$ then encourages higher data flow $\Phi_i(t)$ and so on.

We can suppose that the information flow $\Phi_i(t)$ represents a number of different evolution variants/stories/conclusions that are afterwards analyzed/interpreted/modeled as the information content $I_i(t)$. The result of this resonance principle is maximizing the link between the two hemispheres and achieving the best balance between the syntactical and semantic part of the information.

We can then ask the following questions: Is the resonance principle a fundamental rule of *self-organization*? Are different components organized (structured) through the links so that the knowledge in each of them is maximized? Are such rules of self-organization compatible to minimal energy principle known in physics?

2.4 Information power

From the information flow $\Phi_i(t)$ and information content $I_i(t)$, one can define other quantities. One of the important quantities is the *information power PI*, defined by [22] as the product of information flow $\Phi_i(t)$ times the information content $I_i(t)$. Dimensional analysis easily reveals that the unit of information power is given in Joule per second [J/s] realized thanks to the received bit of information. By introducing the quantity of information power, one can demonstrate that the impact of information is maximized if the received information flow $\Phi_i(t)$ is appropriately processed by the recipient and transformed into the best possible information content $I_i(t)$. If there is a flow of valuable information that the recipient is incapable of processing, the information power level is low. On the other hand, if the recipient is able to make good use of the information flow, but the flow does not carry the needed information, the result is likewise a low level of information power.

We can continue with these ideas even further introducing a phase shift between the information flow $\Phi_i(t)$ and the content $I_i(t)$, thereby arriving at the definition of an active and a reactive information power [23]. We can imagine the *active information power* as a power caused by information, which is transformed directly into concrete physical events. The *reactive information power* represents our emotions, which of course do not perform any work, but which support our decision making. Worth mentioning in this context is a well-known Bible story: the King Solomon proposes to have a child split in halves when two women are fighting over it, but because of her emotions, the real mother would rather give up her child than let it be killed.

An interesting area of the information power is the *perception of time*, which we can imagine as the number of biological events taking place in our bodies with a given frequency. If the information power intake (measured in success events per second) is slower than our biological clock, we have the feeling that time is moving slowly, but if the intake is faster, we have the feeling that time is being well used.

2.5 Information gates

For the sake of simplicity, let us imagine an information subsystem as an input-output information gate shown in Fig. 3.

Between the input ports, the input information content is available, and input information flow enters the system. Between the output ports, it is possible to obtain an output information content, and output information flow leaves the system. The input and output information power PI_{in}, PI_{out} should be assigned to this information gate.

We can furthermore assume that this subsystem is open and is capable of using its surroundings as a source for drawing energy. Kauffman [6] introduced the term *autonomous living agents*, which are characterized by the ability to direct and

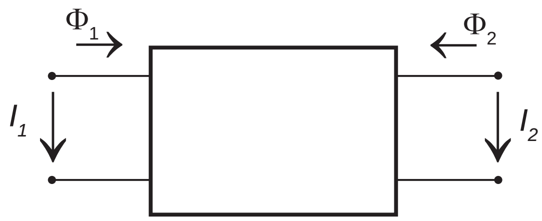


Fig. 3 *Information gate (Φ – information flow of data measured in bits per second, I – information content measured in Joule per bits).*

release energy. Kauffman is also the originator of the idea that the self-organization that is characteristic for living systems is defined by a series of actions leading to the dissemination of macroscopic work.

Let us now examine the input-output information gate we have created. Input quantities can describe purely intellectual operations. The input information content includes our existing knowledge, and the input information flow describes the change of the environment in which our gate operates and the tasks we want to be carried out (target behavior). All of the valuable, long-term information gained in this way can be used for the targeted release of energy, where at the output of the input-output gate, there may be an information content in the order of Joules per bit (or profits in dollars). The output information flow serves as a model to provide such services or knowledge.

The basis of information systems is the ability to interconnect individual information subsystems, or in our case, input-output information gates. It is very easy to imagine the *serial or parallel ordering* of these gates into higher units. A very interesting model is the *feedback of information gates*, because it leads to a nonlinear characteristics, to an information systems defined at the limit of stability and other interesting properties.

In this manner one may define for example *information filters* which are able to select, remove or strengthen a particular component of information.

2.6 Student-teacher interaction

In the context of information systems, it is also necessary to deal with the problem of teaching, because the information subsystem called a teacher may be regarded as a source of information content. The teacher has been preparing this information content for years with respect to both the content as such (optimizing the information content) and its didactic presentation (optimizing the information flow), so that the knowledge can be passed on to a subsystem known as a student. If we assume that the teacher subsystem has greater information content than the student subsystem, after their interconnection, the information flow will lead from the teacher to the student, so that the information content of the two systems will gradually balance out.

The students receive the information flow and increase their information content. If the students are not in a good mood, or if the information flow from the teacher is confused, the students are unable to understand the information received and to process it, so as to increase their information content. With the help of the reactive information power mentioned above, which concerns the emotional aspects of the recipient and the source, i.e. the student and the teacher, it is possible to create a situation where the students' sensitivity to the received information flow is maximized, so that they are able to process it appropriately and transform it into information content. Analogously, the teacher being in a good mood can lead to the creation of better information flow.

The individual components and subsystems of information systems can behave in different ways, and their behavior can be compared to everyday situations in our lives. A characteristic of politicians is their ability to use even a small input of information content to create a large output information flow. They posses the ability to take a small amount of superficially understood content to interpret and explain it to the broadest masses of people. On the other hand, a typical professor might spend years receiving the input information flow and input information content, and within her/his field, he/she may serve as a medium for transmitting a large quantity of output information content. The professor, however, might not spread the content very far, sharing it perhaps only with a handful of enthusiastic students.

It is hard to find an appropriate system to combine the characteristics of the different information subsystems described above, but it is possible to create a group of subsystems (system alliances), where these characteristics can be combined appropriately. In this way, one can model a company or a society of people who together create an information output that is very effective and varied, leading to improved chances for the survival and subsequent evolution of the given group.

Through an appropriate combination of its internal properties, our information alliance can react and adapt to the changing conditions of its surroundings. Survival in alliances thus defined seems more logical and natural than trying for a combination of all necessary processes within the framework of one universal information subsystem. If we have part of an alliance copied or if we have it divided into two more operative groups that will continue developing and do not lose their connection, we arrive at interesting stimuli for the modeling of the natural emergence, spread or extinction of organisms, companies or societies.

3. Wave models of complex information systems

Currently, a number of interesting results have been discovered in the field of quantum information science [4], taking as their basis the foundations of quantum physics and using for modeling of complex systems those principles that do not arise in classical physics, such as entanglement and quantization [14].

The quantum information quantity in bits can be measured, e.g. by von Neumann entropy which measures the amount of uncertainty contained within the density operator also taking into account some wave probabilistic features such as entanglement.

Suppose that the studied system does not possess any quantum features, the von Neumann entropy converges into classical Shannon entropy [37]. Referring to this result the wave probabilistic models can be seen as the extension of classical approach presented in Session 2.

3.1 Two (non-)exclusive observers

Now let us imagine that we are flipping a coin, so that every toss comes out as heads or tails. Someone else, who is assigned the role of an observer, is counting the frequency of the individual coin tosses and is estimating the probability of the phenomenon of it landing heads or tails in a simple manner, by counting the number of times it has fallen as heads or tails in the past, and by dividing that number by the number of the observed or registered tosses. If the observer performs this activity for a sufficient length of time, the resulting probability will be tossing heads fifty percent of the time, while the probability of tossing tails will also be fifty percent, if all of the tosses are done in a proper manner and if the coin has a perfect shape (disk) and uniform density.

Now let us try to extend further this simple example for possible variants involving errors by the observer, and let us imagine what would happen if our observer were imperfect and made errors when observing. The observer, for example, might wear thick glasses and have difficulty telling heads from tails, with the result that from time to time, he/she would incorrectly register a toss as heads or tails, and this would then show up in the resultant probability as a certain error. Because there is only one observer, we automatically, and often even unconsciously, assume that his/ her observations are exclusive. Exclusivity means that when our observer registers a toss of heads, he/she automatically does not register a toss of tails, and to the contrary, when registering a toss of tails, he/she does not at the same time register a toss of heads. Thanks to this property, the sum of the resultant probabilities of heads and tails always equals one hundred percent regardless of the size of the observer's error. The error of the observer shows up only by increasing the probability of one side of the coin, while at the same time lowering the probability of the opposite side by the same value.

Now let us assume that we are observing the same phenomenon of coin tossing, but now with two observers who are not consulting each other about their observations. There might be two persons, one of whom watches for and registers only tosses of heads and the other only tails. Each of our two observers is counting the frequency of tosses of his or her own side of the coin, meaning that they each divide the number of their respective sides of the coin by the total number of tosses. The results are the probabilities of tossing heads or tails, and if both observers work without any errors, the result will be the same as in the case of one observer, except that more people will be participating in getting the result.

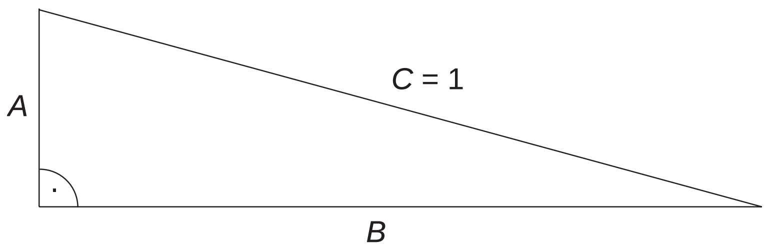
Now let us expand our case with two observers so that it reflects errors on their parts. Just as in the last case, both observers might be wearing thick glasses and might have difficulty telling heads from tails. In the case of two observers, we can no longer assume that their observations are exclusive, because as we said, we are assuming that they are not consulting their observations with each other.

What might happen in this situation? At a given moment, one observer could see a toss of heads registering that phenomenon, and the second observer might independently evaluate the coin toss as tails registering tails. Or the other way round: the first observer will see that the toss was not heads, and the other that the toss was not tails. In that situation, a coin toss is registered, but it is registered neither as heads nor as tails. Logically, as an outcome of these two situations, the sum of the resulting probabilities will not equal the desired one hundred percent, but will be either greater than one hundred percent in the first case, or less than one hundred percent in the second. From a mathematical perspective, this would mean violation of the fundamental law of probabilities, that the sum of the probabilities of all possible phenomena in a complete system must equal one hundred percent.

How can we get around this problem? We can help ourselves by imagining the geometry of a triangle and its use in the theory of probability. Let us first assume, in accordance with Fig. 4, that the triangle is a right-angled triangle, and that the length of the square root of the probability of tossing heads is depicted on the x-axis, the length of the square root of the probability of tossing tails being shown on the y-axis.

Tossing heads:	H	Tossing experiment:	$H, T, T, H, H, T, T, T, H, T$
Tossing tails:	T	Number of registered tossing:	10
P observer registered:	4	O observer registered:	6

$$p(H) = \frac{4}{10} \qquad p(T) = \frac{6}{10} \qquad \beta = \frac{\pi}{2}$$



$$C^2 = 1 = A^2 + B^2 = \left|A + B \cdot \mathrm{e}^{j \cdot \frac{\pi}{2}}\right|^2$$

$$p(H) + p(T) = \left|\sqrt{p(H)} + \sqrt{p(T)} \cdot \mathrm{e}^{j \cdot \frac{\pi}{2}}\right|^2 = 1$$

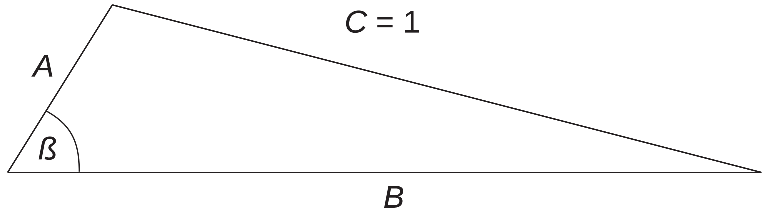
Fig. 4 *A right triangle – in this case of tossing coins, it must be true that 'C' = 1 (i.e. 100%). 'A' is the probability of tossing heads $p(H)$, and 'B' is the probability of tossing tails $p(T)$.*

If we use the Pythagorean theorem that the sum of the squares of the legs equals the square of the hypotenuse, we can say that the length of the hypotenuse of the right triangle in this case must equal one (i.e. 100%). This would correspond to a geometrical interpretation of the required property that the sum of the probabilities of tossing heads and of tossing tails must equal one. At the same time, this geometric analogy characterizes probabilities as squares of the lengths of sides of a triangle. The right angle of the triangle is then an indication of the exclusivity of the observations.

Now let us deal with the geometric interpretation of the errors of our two observers. Under the condition that the length of the hypotenuse of a right triangle must always equal one, we can model the error rates of our observers using the angle between the triangle's legs, so that the square root of the probability determined by the first observer (including his or her errors) will be depicted on the x-axis and the square root of the probability found by the second observer (including that observer's errors) will be depicted on the y-axis. Mathematically, we can apply the law of cosines to whatever kind of a triangle this produces as it is shown in Fig. 5, instead of using the Pythagorean theorem that applies only to exclusive observations resulting in a right-angled triangle.

Tossing heads:	H	Tossing experiment:	$H, T, T, H, H, T, T, T, H, T$
Tossing tails:	T	Number of registered tossing:	10
P observer registered:	2	O observer registered:	5

$$p(H) = \frac{2}{10} \qquad p(T) = \frac{5}{10} \qquad \beta = 1.076$$



$$C^2 = 1 = A^2 + B^2 \pm 2 \cdot A \cdot B \cdot \cos(\beta) = \left|A + B \cdot \mathrm{e}^{j \cdot \beta}\right|^2$$

$$p(H) + p(T) \pm 2 \cdot \sqrt{p(H)} \cdot \sqrt{p(T)} \cdot \cos(\beta) = \left|\sqrt{p(H)} + \sqrt{p(T)} \cdot \mathrm{e}^{j \cdot \beta}\right|^2 = 1$$

Fig. 5 *In this non-right-angled triangle, in our case of coin tosses, 'C' still must equal 1. 'A' represents the probability of tossing heads as registered by the first observer $p(H)$, and 'B' is the probability of tossing tails as registered by the second observer $p(T)$. The angle β models the errors of the observers.*

What does this situation mean, and how can it be interpreted generally? The two observers are independent of each other, without being aware of the fact and without sharing any information with each other. Their (virtual) interconnection is represented geometrically by the angle between the x- and the y-coordinate, representing the mutual imperfection of their observing. The more perfect their observing is, the less they are dependent. In the case of perfect observers, this dependence disappears completely, corresponding geometrically to a right triangle.

Now let us examine the parallel between a signal breakup into harmonic components and the probability theory. Probability values have analogies to energies and can be modeled as the squares of the values assigned to individual phenomena (concrete values). By the square roots of the probability of event phenomena, one may interpret how dominant a given phenomenon is in a random process, or how often the phenomenon occurs. In this conception, phase indicates the degree of structural links between the individual phenomena [32], i.e. by analogy the shift with respect to the defined beginning. This beginning may be a phenomenon with a zero phase, to which we relate all of the structural links of the other phenomena.

Unlike classical information science, where the state of a system, or more precisely, information about its state, is described with the use of a probability function, in quantum information science, the information about the state of the system is described using a wave probabilistic function.

Let us define discrete events A and B of a sample space S, with defined probabilities $P(A)$, $P(B)$. The quantum state $|\psi\rangle$ represents the description of the quantum object given by superposition of these events [3]:

$$|\psi\rangle = \psi(A) \cdot |A\rangle + \psi(B) \cdot |B\rangle\,, \tag{1}$$

with wave probabilistic functions defined as

$$\psi(A) = \alpha_A \cdot \mathrm{e}^{j \cdot \upsilon_A}, \quad \psi(B) = \alpha_B \cdot \mathrm{e}^{j \cdot \upsilon_B}, \tag{2}$$

where $\alpha_A = \sqrt{P(A)}$, $\alpha_B = \sqrt{P(B)}$ are modules, and υ_A, υ_B are the phases of a wave probabilistic function. In accordance with the general principle, we can see that we obtain the classical theory of probability by raising the complex wave function to the second power, whereby we automatically lose the phase characteristic of our model.

What do these ideas have to do with quantum physics? In the case of quantum physics, there is a definite model of behavior of a studied system, which we affect by our method of measurement. This means that the result of the measurement is not a description of the original system, but of a new behavior of the system influenced by our measuring. We get something that can be compared with our observer with thick glasses, i.e. a model that is dependent on the observer. In order to find a model of the behavior of the original system (without intervention by measuring), we must eliminate the error of the observer, that is to say, we must introduce phase parameters to our model that correct the intervention of the method of measurement.

3.2 Two binary quantum subsystems

Let us define the first quantum binary subsystem and suppose it can reach two values $A = 0$ and $A = 1$. The second quantum binary subsystem can achieve values $B = 0$ and $B = 1$. We assume the phase to be the linear function of quantized phase $m \cdot \Delta$. The wave probability function takes the form $\psi \propto \mathrm{e}^{j \cdot k \cdot (\theta + n \cdot 360)}$, where the symbol $\propto$ means equality up to the normalization factor and 'j' represents an imaginary unit. The phase function must achieve single-valuedness also for the phases $(\Delta + 2 \cdot \pi \cdot k)$ where k is an integer [20].

Additional assumption is distinguishability of each system [4]. Mathematically we arrive at following wave probabilistic functions:

$$\psi(A=0) = \sqrt{P(A=0)}, \quad \psi(A=1) = \sqrt{P(A=1)} \cdot \mathrm{e}^{j \cdot m \cdot (\Delta + 2 \cdot k \cdot \pi)}, \tag{3}$$

$$\psi(B=0) = \sqrt{P(B=0)}, \quad \psi(B=1) = \sqrt{P(B=1)} \cdot \mathrm{e}^{j \cdot m \cdot (\Delta + 2 \cdot k \cdot \pi)}. \tag{4}$$

We can suppose that observer No.1 monitors the state $A = 0$ and observer No.2 the state $B = 1$. The probabilities union that $A = 0$ or $B = 1$ is given as:

$$\begin{aligned} P((A=0) \cup (B=1)) &= \left| \sqrt{P(A=0)} + \sqrt{P(B=1)} \cdot \mathrm{e}^{j \cdot m \cdot (\Delta + 2 \cdot k \cdot \pi)} \right|^2 = \\ &= P(A=0) + P(B=1) + 2 \cdot \sqrt{P(A=0) \cdot P(B=1)} \cdot \cos(m \cdot (\Delta + 2 \cdot k \cdot \pi)) \end{aligned} \tag{5}$$

which is the quantum equivalent of the classical well-known probabilistic rule:

$$P((A=0)\cup(B=1)) = P(A=0) + P(B=1) - P((A=0)\cap(B=1)). \quad (6)$$

The quantum rule (5) enables both a negative and a positive sign due to a phase parameter. It is evident that the intersection of probabilities in quantum world can be also negative $P((A=0)\cap(B=1)) < 0$ despite of the fact that probabilities $P(A=0) \geq 0, P(B=1) \geq 0$ are positive.

The quasi-spin was firstly introduced in [22]. If the quasi spin is integer $m \in \{0, \pm 1, \pm 2, \pm 3, \ldots\}$ we can guarantee the positive sign in (5). Such quantum subsystems are called the information bosons. The information fermions are characterized with half-integer quasi-spin $m \in \left\{0, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots\right\}$ and we must admit the negative sign in (5). We can also deduce the information quarks analogous to quantum physics with its special properties [24].

3.3 Quantum entanglement

One of the most remarkable phenomena of quantum physics is quantum entanglement. This phenomenon has no parallel in classical physics, and it cannot even be generated using classical methods, because it uses the principle of the mutual disruption of certain states of a system thanks to wave probability functions.

We can take a closer look at this phenomenon using our simple example with coin tossing. Suppose we have two parallel systems, and that the outcome of each system is either throwing heads or tails. If the probability from measuring the first system is a fifty-percent chance of throwing heads or a fifty-percent chance of throwing tails, the output from measurement of the second system is determined entirely by the value of measurement of the first system. In other words, the output of the second system is 100-percent entangled with the output of the first system, because if the output of the first system is heads, for the second system we will most certainly, actually with a 100-percent probability, get a reading of tails. And vice-versa: if the output of the first system is tails, the output of the second system will definitely be heads. This conclusion applies regardless of the distance between the two systems.

Let us define the above mentioned example through joint probability

$$\begin{aligned} &P((A=0)\cup(B=1)) = \\ &\quad = P(A=0) + P(B=1) + 2\cdot\sqrt{P(A=0)\cdot P(B=1)}\cdot\cos(\varphi), \end{aligned} \quad (7)$$

where φ is the phase difference between wave functions $\psi(A=0)$ and $\psi(B=1)$. Let us suppose now that

$$P((A=0)\cup(B=1)) = 0. \quad (8)$$

This case can occur for the following values of φ:

$$\varphi = a\cos\left(-\frac{1}{2}\cdot\frac{P(A=0)+P(B=1)}{\sqrt{P(A=0)\cdot P(B=1)}}\right). \quad (9)$$

If, for example, $P(A=0) = P(B=1) = 1/2$, then $\varphi = \pi$ represents the 100-percent entanglement.

As a result of the entanglement (8), we can write that the following events will surely happen:

$$P\left((A=1)\cap(B=0)\right)=1. \tag{10}$$

We can also start with the following probability, instead of (8):

$$P\left((A=1)\cup(B=0)\right)=0. \tag{11}$$

Then the entanglement yields into

$$P\left((A=0)\cap(B=1)\right)=1. \tag{12}$$

Measuring the first quantum object (the probability of measuring event 0 is 1/2 and the probability of measuring 1 is also 1/2) fully determines the value which will be measured on the second object. Eqs. (10) and (12) yield the well-known Bell state introduced in [37], which is used in many applications, for example in quantum teleportation, quantum cryptography, etc.

3.4 Quantum processes

Let us imagine that thanks to a complex wave probability function, a situation may arise when we shall be monitoring the probability of the union of several phenomena, i.e. that either the first phenomenon will occur, *or* the second will occur, *or* the third will not occur etc., and that this probability works out to equal zero. Naturally, this situation cannot arise under the classical theory of probability, because their probabilities are merely added together, and at the most, repeating overlaps of phenomena are subtracted. In the newly introduced area of complex wave probability functions, it can also occur, through the influence of the existence phases, the subtracting of probabilities, and under certain conditions it is possible to find such a constellation of phenomena, that their union works out to zero probability. This, however, automatically means that the inversion phenomenon (intersection) for the given union (in our case, this inversion phenomenon would mean that the first phenomenon does not occur, *and* at the same time the second phenomenon does not occur, *and* the third phenomenon does occur) will occur with 100% probability, regardless of how the phenomena are arranged spatially.

Quantum entanglement is caused by the resonance of complex wave functions. Among the ways this resonance manifests itself is that thanks to it we arrive from a purely probabilistic world to a completely deterministic world, where there is a disruption of the probabilistic characteristics of various phenomena, and the links between the entangled phenomena become purely deterministic events that even show up in different places (generally even at different times), and for that reason they are also often designated as spatial (or generally temporo-spatial) distributed system states. Similarly, one may arrive at the conclusion that thanks to the principle of resonance, selected (temporo-spatial) distributed states absolutely cannot occur in parallel, and this leads to an analogy with the Pauli exclusion principle.

The selection of a group entangled states can, of course, have a probabilistic character, as long as the entanglement is not one hundred percent. This means that parallel behavior occurs only with a certain probability, and this leads to the idea of the selection of one variant according to the given probability function.

In reference [37], we read that the behavior of entangled states is very odd. Firstly, it spreads rapidly among various phenomena, where for this spreading it makes use of a property known as entanglement swapping. Here is a simple example of this behavior. If we have four phenomena, the first and second being entangled,the third and fourth phenomenon being entangled as well, then as soon as it comes to an entanglement between the first and third phenomenon, the second and fourth are also entangled, without any information being exchanged between them. Not- withstanding that those phenomena can be spatially quite remote from each other.

3.5 Quantum information gate

With respect to above mentioned electrical-information analogies we can also define the wave information flow and the wave information content such as the wave probabilistic functions (for the sake of simplicity we suppose that all quantities are time independent):

$$\psi_\Phi(x) = |\psi_\Phi(x)| \cdot \mathrm{e}^{j \cdot \nu_\Phi(t)}, \qquad \psi_I(x) = |\psi_I(x)| \cdot \mathrm{e}^{j \cdot \nu_I(x)}. \tag{13}$$

Referring to the above results we can also redefine the information power on the level of wave probabilistic functions in the following way [4]:

$$PI(x) = |\psi_\Phi(x)| \cdot |\psi_I(x)| \cdot \cos(\nu_I(x) + \nu_\Phi(x)). \tag{14}$$

Let us define quantities as follows:

$$\psi_\Phi = \alpha_{\Phi,1} \cdot |\Phi_1\rangle + \alpha_{\Phi,2} \cdot |\Phi_2\rangle + \dots + \alpha_{\Phi,N} \cdot |\Phi_N\rangle \tag{15}$$

$$\psi_I = \alpha_{I,1} \cdot |I_1\rangle + \alpha_{I,2} \cdot |I_2\rangle + \dots + \alpha_{I,N} \cdot |I_N\rangle \tag{16}$$

where $\Phi_1, \dots, \Phi_N$ and $I_1, \dots, I_N$ are possible values of information flow and information content, respectively. Complex parameters $\alpha_{\Phi,1}, \dots, \alpha_{\Phi,N}$ and $\alpha_{I,1}, \dots, \alpha_{I,N}$ represent wave probabilities taking into account both probability of falling relevance flow/content value together with their mutual dependences [15].

The information power can be expressed through wave probabilistic functions as follows (under assumption of distinguishability):

$$\begin{aligned}\psi_{PI} = \psi_\Phi \otimes \psi_I = \alpha_{\Phi,1} \cdot \alpha_{I,1} \cdot |\Phi_1, I_1\rangle + \dots + \alpha_{\Phi,1} \cdot \alpha_{I,N} \cdot |\Phi_1, I_N\rangle + \dots \\ \dots + \alpha_{\Phi,N} \cdot \alpha_{I,1} \cdot |\Phi_N, I_1\rangle + \dots + \alpha_{\Phi,N} \cdot \alpha_{I,N} \cdot |\Phi_N, I_N\rangle\end{aligned} \tag{17}$$

where symbol $\otimes$ means Kronecker operation [15, 16] for vectors transformed into multiplication, each i,j-th component $|\Phi_i, I_j\rangle$ represents particular value of information power that characterizes the falling/measuring of the information flow Φ_i and information content I_j.

Multiplication of different combinations of the information flows and contents $|\Phi_i, I_j\rangle$, $|\Phi_k, I_l\rangle$ can achieve the same (or similar) information power K_r,

$$\Phi_i \cdot I_j \approx \Phi_k \cdot I_l \approx K_r. \tag{18}$$

It can be seen that interferences of wave probabilities can emerge and wave resonances among the wave parameters are possible as well. Finally, an information power in renormalized form can be expressed as:

$$\psi_{PI} = \beta_1 \cdot |K_1\rangle + \beta_2 \cdot |K_2\rangle + \cdots + \beta_r \cdot |K_r\rangle + \dots . \tag{19}$$

This approach yields to the resonance principle between the received/transmitted information flow and information content with respect to our preferences. It enables modeling deep perception and new soft systems categories both for input/output parameters of each quantum information gate.

It is supposed that each quantum information gate has its wave input/output information flow Φ_i and content I_j. With respect to this statement we can, therefore, define the wave input/output information power $PI_{\text{in}}, PI_{\text{out}}$ assigned to such a gate.

3.6 Two (non-)distinguished quantum subsystems

Let us have two quantum subsystems A, B described by wave probabilistic functions $\psi_A(.)$, $\psi_B(.)$. First of all, we suppose that we are able to distinguish between A and B quantum subsystems. Let us assign features (e.g. a special functionality, a set of parameters or a part of a subsystem) p_1 or p_2 to them. The final quantum system is represented by following wave probabilistic function:

$$\psi_{A,B}(p_1, p_2) = \psi_A(p_1) \cdot \psi_B(p_2) . \tag{20}$$

In case we are not able to assign the right feature to the given subsystems A or B we must apply the principle of quantum indistinguishability [4]. It means we have to take into account all variants of possible arrangements:

$$\psi_{A,B}(p_1, p_2) = \psi_A(p_1) \cdot \psi_B(p_2) \pm \psi_A(p_2) \cdot \psi_B(p_1) , \tag{21}$$

where $\pm$ characterizes the symmetry or non-symmetry of both variants (information bosons or fermions).

Let us suppose that we have generalized “gravitation energy” between our two subsystems $U_{A,B}(p_1, p_2)$. How much energy will be used to connect A and B under the condition of quantum indistinguishability? From (21) we can compute the probability density:

$$\rho(p_1, p_2) = [\psi_A(p_1)]^2 \cdot [\psi_B(p_2)]^2 \pm 2 \cdot \psi_A(p_1) \cdot \psi_B(p_2) \cdot \psi_A(p_2) \cdot \psi_B(p_1) + \\ + [\psi_A(p_2)]^2 \cdot [\psi_B(p_1)]^2 . \tag{22}$$

The mean value of connection energy is given:

$$\bar{U}_{A,B} \approx C_{A,B} \pm X_{A,B}, \tag{23}$$

where $C_{A,B}$ is a classical energy integral and $X_{A,B}$ is the exchange integral – a consequence of quantum indistinguishability. $C_{A,B}$ and $X_{A,B}$ can be computed using (22) under symmetry condition [14]:

$$C_{A,B} = \int\limits_{V_1}\int\limits_{V_2} \left[\psi_A\left(p_1\right)\right]^2 \cdot \left[\psi_B\left(p_2\right)\right]^2 \cdot U_{A,B}\left(p_1, p_2\right) \,\mathrm{d}p_1 \,\mathrm{d}p_2, \tag{24}$$

$$X_{A,B} = \int\limits_{V_1}\int\limits_{V_2} \psi_A\left(p_1\right) \cdot \psi_B\left(p_2\right) \cdot \psi_A\left(p_2\right) \cdot \psi_B\left(p_1\right) \cdot U_{A,B}\left(p_1, p_2\right) \,\mathrm{d}p_1 \,\mathrm{d}p_2. \tag{25}$$

We can mark the distance between subsystems A and B as $R = |p_1 - p_2|$. Then Eq. (22) with minus sign represents the binding in the system as a whole (in analogy with hydrogen atom in physics).

4. Features of complex systems

4.1 Self-organization principles

Let us deal with three subsystems A, B, C not admitting any negative probabilities $P(A)$, $P(B)$ or $P(C)$, which means that the subsystems can only store/carry energy (they do not exhaust it). Further on, we suppose that all the three subsystems fulfil the normalization condition:

$$P\left(A \cup B \cup C\right) = 1. \tag{26}$$

As there is no link between either the subsystems A and C or between the subsystems B and C we must admit with respect to (5) both positive and negative joint probability $P\left(A \cap B\right)$ – information bosons or fermions:

$$P\left(A \cup B \cup C\right) = P\left(A\right) + P\left(B\right) \pm P\left(A \cap B\right) + P\left(C\right) \equiv 1. \tag{27}$$

We can write the Eq. (27) in more universal wave probabilistic form:

$$P\left(A \cup B \cup C\right) = P\left(A\right) + P\left(B\right) + \sqrt{P\left(A\right) \cdot P\left(B\right)} \cdot \cos\left(\varphi\right) + P\left(C\right) \equiv 1, \tag{28}$$

where φ is the phase difference between wave functions $\psi\left(A\right)$ and $\psi\left(B\right)$.

Generally, we can define also one-directional links [29] (phase φ characterizes the source of the link):

$$P\left(A \cup B \cup C\right) = P\left(A\right) + P\left(B\right) + \sqrt{P\left(A\right)} \cdot \cos\left(\varphi\right) + P\left(C\right) \equiv 1, \tag{29}$$

$$P\left(A \cup B \cup C\right) = P\left(A\right) + P\left(B\right) + \sqrt{P\left(B\right)} \cdot \cos\left(\varphi\right) + P\left(C\right) \equiv 1. \tag{30}$$

In case there is no link between A and B, the energy assigned to the probability $(1 - P\left(C\right))$ is distributed between A and B,

$$P\left(A\right) + P\left(B\right) = 1 - P\left(C\right). \tag{31}$$

If we start to model a positive link between A and B (co-operation model characterized by classical probabilistic rule with negative sign of $P\left(A \cap B\right)$ in (27)) we can write

$$P\left(A\right) + P\left(B\right) = 1 - P\left(C\right) + P\left(A \cap B\right). \tag{32}$$

It is evident that the right side of this form is increased. It means that both A and B can gain additional energy due to "the common co-operation principle" at the expense of $P(C)$. For available maximum $P(A) = 1$ and $P(B) = 1$ the link must achieve $P(A \cap B) = 1$ and $P(C) = 0$.

A negative link between A and B can also bring a negative influence modelled by a positive sign of $P(A \cap B)$ in (27). It means that the negative link yields to weakening of both subsystems A and B and to strengthening of the subsystem C. The minimum $P(A) = 0$ and $P(B) = 0$ is fulfilled for $P(A \cap B) = P(C) - 1$. If subsystem C is able to use the lost energy from A and B than such situation is characterized by $P(A \cap B) = 0$ and $P(C) = 1$. If not, negative value of $P(A \cap B)$ means the energy dissipation into system environment. In case $P(C) = 0$ all energy assigned to the subsystems A and B is dissipated into the environment and so $P(A \cap B) = -1$.

The positive and negative links among different subsystems create the emergent behaviour known in theory of complex systems. The more subsystems the more links among them and so the more emergencies that have significant influence to the modelled system as a whole.

Self-organization rules should be explained through the probability (energy) maximization principle. We can search for (positive or negative) links among different subsystems to maximize simultaneously each subsystem (egoistic behaviour) $P(A), P(B), P(C)$ and also the system as a whole (group/alliance behaviour [40]) $P(A) + P(B) + P(C)$. Various criteria for optimization can be studied, e.g. tuning parameters of links for optimal distributions of energies within the complex system. A new model is likely to be formed introducing the natural driving force yielding to the creation of different system structures (organization schemes, unexpected new links, etc.).

4.2 Interference principles

Many complex systems are typically characterized by a high level of redundancies. The surrounding complex reality can be modelled either by very complicated model or approximated by a set of many different and often overlapping easier models which represent different pieces of knowledge.

Wave probabilistic models could be used to set up final behavior of complex system. Phase parameters can compensate overlapping information among models as it was firstly presented in [18]. Feynman rule [4, 5] says that all paths (or in our case each of the models) contributes to a final amplitude (or in our case to a final model) by its amplitude with different phase.

In classical examples the more models the more possible trajectories of the future complex system behavior. This problem is mentioned in literature as "the curse of dimensionality". But for wave probabilistic models some trajectories could be due to phase parameters mutually canceled up and others, by contrast, strengthened. If we take a sum of all trajectories assigned to all wave models this sum can converge into "right" trajectory of the complex system. With respect to Feynman path diagram, the more available models could not note the complexity increase. I would like to show this principle on the following illustrative example.

Let us have three binary subsystems A, B, C characterized by wave probabilities $\psi(A=0)$, $\psi(A=1)$, $\psi(B=0)$, $\psi(B=1)$, $\psi(C=0)$ and $\psi(C=1)$. The whole quantum system can be described (under the distinguishability assumption) as

$$\psi = \psi(A) \otimes \psi(B) \otimes \psi(C) = \gamma_{0,0,0} \cdot |000\rangle + \gamma_{0,0,1} \cdot |001\rangle + \cdots + \gamma_{1,1,1} \cdot |111\rangle \,, \quad (33)$$

where $\gamma_{i,j,k} = \psi(A=i) \cdot \psi(B=j) \cdot \psi(C=k)$ is the wave probability assigned for $i,j,k \in \{0,1\}$. It is evident that eight possible quantum processes are possible.

We can imagine that due to the interferences of the wave probabilistic functions $\gamma_{i,j,k}$ only two final processes $|000\rangle$ and $|111\rangle$ can take place as written bellow:

$$\gamma_{0,0,0} + \gamma_{1.1.1} = 1, \qquad \gamma_{0,0,1} + \gamma_{0.1.0} + \gamma_{1.0.0} + \gamma_{0,1,1} + \gamma_{1,0,1} + \gamma_{1,1,0} = 0, \quad (34)$$

even though we can separately measure all eight variants with probability $|\gamma_{i,j,k}|^2$ in each of the systems.

The presented illustrative example can be extended into more complex time-varying systems but the basic principles are the same. The whole is more than the sum of different pieces because it can possess new emergent features caused by interferences of its parts.

4.3 Identity principles

Let us define two binary subsystems A and B characterized by the wave probabilities $\psi(A=0)$, $\psi(A=1)$, $\psi(B=0)$ and $\psi(B=1)$. In [20, 21] the product probabilistic rule for two wave functions was defined as

$$P((A=1) \cap (B=1)) = \frac{1}{2} \cdot \left[\psi^*(A=1) \cdot \psi(B=1) + \psi(A=1) \cdot \psi^*(B=1)\right], \quad (35)$$

where the symbol ψ^* expresses a complex conjugate of ψ. This equation is in compliance with the non-distinguishability principle because the replacement between A and B has no influence on the probability $P((A=1) \cap (B=1))$.

We can imagine that A represents the real subsystem and B its external image or in other words: how this subsystem is perceived/accepted by its environment/surroundings. This quality was firstly introduced by [38] as a subsystem identity.

It is reasonable to suppose that the surroundings spend no energy to make changes of the subsystem A, which means

$$|\psi(A=1)| = |\psi(B=1)| \,. \quad (36)$$

In case the surroundings fully accept the subsystem A both subsystems A and B are identical (they have the same phases) and we can rewrite (35) as a standard Copenhagen interpretation form [4],

$$P(A=1) = |\psi(A=1)|^2 \,. \quad (37)$$

The acceptance of the subsystem A by its surroundings (modeled by its image B) can be differentiated by phase parameters. We note the phase difference between $\psi(A=1)$ and $\psi(B=1)$ as $\Delta\varphi$. Then, (35) can be given as

$$P((A=1) \cap (B=1)) = |\psi(A=1)|^2 \cdot \cos(\Delta\varphi) \,. \quad (38)$$

There are many variants $\Delta\varphi$ for the subsystems identity modelling available. The full acceptance is modelled by $\Delta\varphi = 0$. The phase difference $\Delta\varphi = \pi$ represents a negative acceptance (the surroundings are blind to it) that yields to the negative sign of $P\left((A=1)\cap(B=1)\right)$.

4.4 Hierarchical networks

In many practical applications of complex system's analyses there is a demand for modelling of hierarchical networks as it is shown in Fig. 6. We can assume that the first layer subsystems $A1$, $A2$, $A3$ and $A4$ play the key roles (system's genetic code [39]) represented by the probabilities $P(A1), P(A2), P(A3)$ and $P(A4)$. The second and third layer is responsible for co-ordination activities: $B1$ co-ordinates $A1$ and $A2, B2$ co-ordinates $A3$ and $A4$ and $C1$ is responsible for collaboration between $B1$ and $B2$.

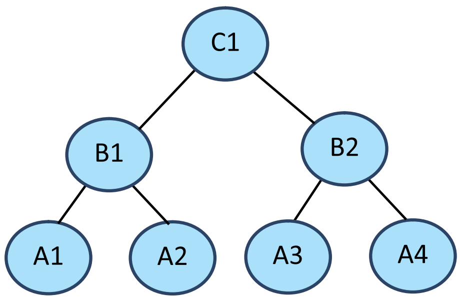


Fig. 6 *Hierarchical model of complex system.*

Let us apply wave probabilistic approach to the network in Fig. 6. We can define wave probabilities assigned to the first layer's functions:

$$\psi\left(A1\right) = \sqrt{P\left(A1\right)} \cdot \mathrm{e}^{j \cdot \varphi_1}, \quad \psi\left(A2\right) = \sqrt{P\left(A2\right)} \cdot \mathrm{e}^{j \cdot \varphi_2}, \tag{39}$$

$$\psi\left(A3\right) = \sqrt{P\left(A3\right)} \cdot \mathrm{e}^{j \cdot \varphi_3}, \quad \psi\left(A4\right) = \sqrt{P\left(A4\right)} \cdot \mathrm{e}^{j \cdot \varphi_4}. \tag{40}$$

The whole system can be described as follows:

$$\begin{gathered}\psi = \psi\left(A1\right)\cdot\psi\left(A2\right)\cdot\psi\left(A3\right)\cdot\psi\left(A4\right) = P\left(A1\right)+P\left(A2\right)+P\left(A3\right)+P\left(A4\right)+\\ +2\sqrt{P\left(A1\right)\cdot P\left(A2\right)}\cdot\cos\left(\varphi_2-\varphi_1\right)+2\sqrt{P\left(A1\right)\cdot P\left(A3\right)}\cdot\cos\left(\varphi_3-\varphi_1\right)+\\ +2\sqrt{P\left(A1\right)\cdot P\left(A4\right)}\cdot\cos\left(\varphi_4-\varphi_1\right)+2\sqrt{P\left(A2\right)\cdot P\left(A3\right)}\cdot\cos\left(\varphi_3-\varphi_2\right)+\\ +2\sqrt{P\left(A2\right)\cdot P\left(A4\right)}\cdot\cos\left(\varphi_4-\varphi_2\right)+2\sqrt{P\left(A3\right)\cdot P\left(A4\right)}\cdot\cos\left(\varphi_4-\varphi_3\right).\end{gathered} \tag{41}$$

Based on the Eq. (41) we can see that the links (hierarchical co-ordinations) could be positive or negative with respect to phase parameters φ_1, φ_2, φ_3, φ_4. We can introduce wave probabilities assigned to the components $B1, B2$ and C :

$$\psi\left(B1\right) = 2\sqrt{P\left(A1\right)\cdot P\left(A2\right)}\cdot\cos\left(\varphi_2-\varphi_1\right), \tag{42}$$

$$\psi\,(B2) = 2\sqrt{P\,(A3)\cdot P\,(A4)}\cdot\cos\left(\varphi_4-\varphi_3\right), \tag{43}$$

$$\begin{aligned}\psi\,(C) = {} & 2\,\sqrt{P\,(A1)\cdot P\,(A3)}\cdot\cos\left(\varphi_3-\varphi_1\right)+\\ & 2\,\sqrt{P\,(A1)\cdot P\,(A4)}\cdot\cos\left(\varphi_4-\varphi_1\right)+\\ & 2\,\sqrt{P\,(A2)\cdot P\,(A3)}\cdot\cos\left(\varphi_3-\varphi_2\right)+\\ & 2\,\sqrt{P\,(A2)\cdot P\,(A4)}\cdot\cos\left(\varphi_4-\varphi_2\right).\end{aligned} \tag{44}$$

Optimal management of hierarchical networks consists of the identification of best arrangement of all subsystems (amplitudes and phases of all components). The coordination process tries to eliminate negative links while supporting the positive links in such a way that the working components $A1, A2, A3$ and $A4$ gain as many probabilities (proportional to the energies) as possible.

The presented example can be extended into more sophisticated networks with many links and more complicated component arrangements. The structure of the network can also cover serial, parallel or feedback ordering of its components. In the future research the methodology similar to Feynman diagrams [4, 5] could be prepared as a part of the wave system theory. Obviously, some components are non-distinguishable which means that all their combinations must be taken into account [20]. For such cases the system's structure optimization may yield into very interesting results. It appears that network's analyze through wave probabilities can get further into the complex system theory which can model features, as emergencies or self-organization.

4.5 Complex quantum systems

The complex quantum systems were analyzed by a methodology that enables to order different basic gates in the same way as in the systems theory. A general description of a quasi-stationary quantum system [20] can be defined as follows:

$$\begin{aligned}\begin{bmatrix}\gamma_1\,(t+1)\\ \gamma_2\,(t+1)\\ .\\ \gamma_n\,(t+1)\end{bmatrix} = k_1(t)\cdot\Bigg\{ & \begin{bmatrix}a_{1,1} & a_{1,2} & . & a_{1,n}\\ a_{2,1} & . & . & .\\ . & . & . & .\\ a_{n,1} & a_{n,2} & . & a_{n,n}\end{bmatrix}\cdot\begin{bmatrix}\gamma_1(t)\\ \gamma_2(t)\\ .\\ \gamma_n(t)\end{bmatrix}+\\ & +\begin{bmatrix}b_{1,1} & b_{1,2} & . & b_{1,n}\\ b_{2,1} & . & . & .\\ . & . & . & .\\ b_{n,1} & b_{n,2} & . & b_{n,n}\end{bmatrix}\cdot\begin{bmatrix}\beta_1(t)\\ \beta_2(t)\\ .\\ \beta_n(t)\end{bmatrix}\Bigg\},\end{aligned} \tag{45}$$

$$\begin{aligned}\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\\ .\\ \alpha_n(t)\end{bmatrix} = k_2(t)\cdot\Bigg\{ & \begin{bmatrix}c_{1,1} & c_{1,2} & . & c_{1,n}\\ c_{2,1} & . & . & .\\ . & . & . & .\\ c_{n,1} & c_{n,2} & . & c_{n,n}\end{bmatrix}\cdot\begin{bmatrix}\gamma_1(t)\\ \gamma_2(t)\\ .\\ \gamma_n(t)\end{bmatrix}+\\ & +\begin{bmatrix}d_{1,1} & d_{1,2} & . & d_{1,n}\\ d_{2,1} & . & . & .\\ . & . & . & .\\ d_{n,1} & d_{n,2} & . & d_{n,n}\end{bmatrix}\cdot\begin{bmatrix}\beta_1(t)\\ \beta_2(t)\\ .\\ \beta_n(t)\end{bmatrix}\Bigg\},\end{aligned} \tag{46}$$

where the matrices **A**, **B**, **C**, **D** are LTI (Linear Time Invariant) evolution $n \times n$ matrices and n-valued discrete input time series observed in the time instant t in the wave probabilistic form can be expressed:

$$|\zeta, t\rangle = \beta_1(t) \cdot |I_1\rangle + \cdots + \beta_n(t) \cdot |I_n\rangle \,, \tag{47}$$

where $I_1, I_2, \ldots, I_n$ is the set of possible values that appears in the studied process and $\beta_1(t), \beta_2(t), \ldots, \beta_n(t)$ is the vector of complex parameters assigned into the input probabilistic discrete values normalized as follows:

$$|\beta_1(t)|^2 + |\beta_2(t)|^2 + \cdots + |\beta_n(t)|^2 = 1. \tag{48}$$

In the same way we can define the n-valued output probabilistic discrete process/signal:

$$|\psi, t\rangle = \alpha_1(t) \cdot |I_1\rangle + \cdots + \alpha_n(t) \cdot |I_n\rangle \,, \tag{49}$$

with normalized complex parameters $\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)$.

The constants $k_1(t)$, $k_2(t)$ guarantee the normalization conditions in each time instant t, and complex parameters $\gamma_1(t), \gamma_2(t), \ldots, \gamma_n(t)$ represent the state-space process/signal (inner parameters):

$$|\zeta, t\rangle = \gamma_1(t) \cdot |I_1\rangle + \cdots + \gamma_n(t) \cdot |I_n\rangle \tag{50}$$

The most general model can also be defined through the time varying evolution matrices $\mathbf{A}(t), \mathbf{B}(t), \mathbf{C}(t), \mathbf{D}(t)$. Because of the difficulty in time evolution modelling of matrices $\mathbf{A}(t), \mathbf{B}(t), \mathbf{C}(t), \mathbf{D}(t)$, we can preferably introduce the quasi-stationary model and use an approach known in the dynamic system theory, e.g. exponential forgetting [27].

Let us present illustrative example of a quantum system with two repeated eigenvalues and one distinct eigenvalue $\lambda_1 = -\frac{1}{2}$, $\lambda_2 = -\frac{1}{2}$, $\lambda_3 = -1$, as follows:

$$\begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \\ \alpha_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.7 & 0.9 & -0.2 \end{bmatrix} \begin{bmatrix} \alpha_1(t-1) \\ \alpha_2(t-1) \\ \alpha_3(t-1) \end{bmatrix}, \qquad \begin{bmatrix} \alpha_1(1) \\ \alpha_2(1) \\ \alpha_3(1) \end{bmatrix} = \begin{bmatrix} 0.5477 \\ 0.5477 \\ 0.5477 \end{bmatrix}. \tag{51}$$

The initial values were chosen as

$$\alpha_1(1) = \alpha_2(1) = \alpha_3(1) = \frac{1}{\sqrt{3}}, \tag{52}$$

so that the initial probabilities were equal to

$$p_1(1) = p_2(1) = p_3(1) = \frac{1}{3}. \tag{53}$$

Fig. 7 presents the time evolution of the probabilities assigned to each state. The analysis shows that the evolution of probabilities converges into the final values: $p_1 = 0.42$, $p_2 = 0.32$ and $p_3 = 0.26$.

We can extend the quantum modeling from the set of n-values to the set of n multi-models [18]. Let the sequence with m output values $Y_z, z \in \{1, 2, \ldots, m\}$ be represented by a set of n-models $P\left(Y_z | H_i\right), i \in \{1, 2, \ldots, n\}$ and let the models

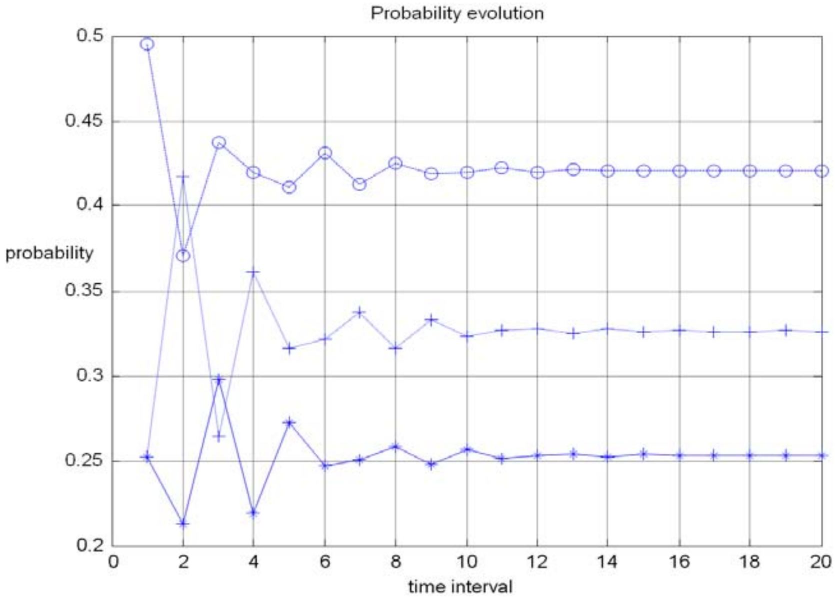


Fig. 7 *Evolution of probabilities assigned to three states – ($*$ marks the state p_1, $+$ marks the state p_2, $\circ$ marks the state p_3).*

be changed over with probability $P(H_i)$. Then, according to well-known Bayes' formula, the probability of z-th output value can be computed as follows:

$$P(Y_z) = \sum_{i=1}^{n} P(Y_z | H_i) \cdot P(H_i). \tag{54}$$

Equation (54) holds only if we know both probabilities $P(H_i)$ and the model components $P(Y_z| H_i)$, $i \in \{1, 2, \dots, n\}$.

Model components $P(Y_z| H_i)$ represent, in our approach, the partial knowledge of the modeled system. In practical situations the number of model components n is finite and is often chosen as a predefined set of multi-model components $P(Y_z | H_i, C)$ where C denotes that the model component is conditioned by the designer decision (letter C meaning the context transition). The probabilities $P(H_i)$ mean the combination factors of the model components.

In the case where the real model components $P(Y_z | H_i)$ are the same as the designer's models $P(Y_z | H_i, C)$, the Eq. (54) is fulfilled. In other cases, the Bayes's formula must be changed so that the designer's decision is corrected:

$$\begin{aligned} P(Y_z) = &\sum_{i=1}^{n} P(Y_z \mid H_i, C) \cdot P(H_i) + \\ &+ 2 \sum_{k<L} \sqrt{P(Y_z \mid H_k, C) \cdot P(H_k) \cdot P(Y_z \mid H_L, C) \cdot P(H_L)} \cdot \lambda_{k.L}^{(z)}, \end{aligned} \tag{55}$$

where coefficients $\lambda_{k,L}^{(z)} = \cos\left(\beta_{k,L}^{(z)}\right)$ are normalized statistic deviations that could be computed by algorithm [18]. The form (55) represents multidimensional Law of cosines that, for the two-dimensional case, could be written as $a^2 = b^2 + c^2 + 2bc\cos(\varphi)$, where φ is the angle between the sides b and c [7, 8].

The probability of z-th output value $P(Y_z)$ can be characterized by a complex parameter $\psi(Y_z)$ with the following properties:

$$P(Y_z) = |\psi(Y_z)|^2, \tag{56}$$

$$\psi(Y_z) = \sum_{i=1}^{n} \psi_i(Y_z), \tag{57}$$

$$\psi_i(Y_z) = \sqrt{P(Y_z | H_i, C) \cdot P(H_i)} \cdot \mathrm{e}^{j \cdot \beta_z(i)}. \tag{58}$$

Because equations (56), (57), and (58) are independent on the selection of the models $P(Y_z \mid H_i, C)$, $i \in \{1, 2, \ldots, n\}$, these models can be chosen in advance to cover a whole range of the probabilistic area (universal models). The multi-models parameters $P(H_i)$ and $\beta_z(i)$ can be estimated from a real data sample (such as amplitude and phase in Fourier transform) to model real system dynamics.

The amplitude and phase representation of multi-models can be expressed as in Fig. 8 (the number of a priori models is selected to 4) where amplitudes define the probability of model occurrence and the phases represent the model composition rule to catch original dynamics.

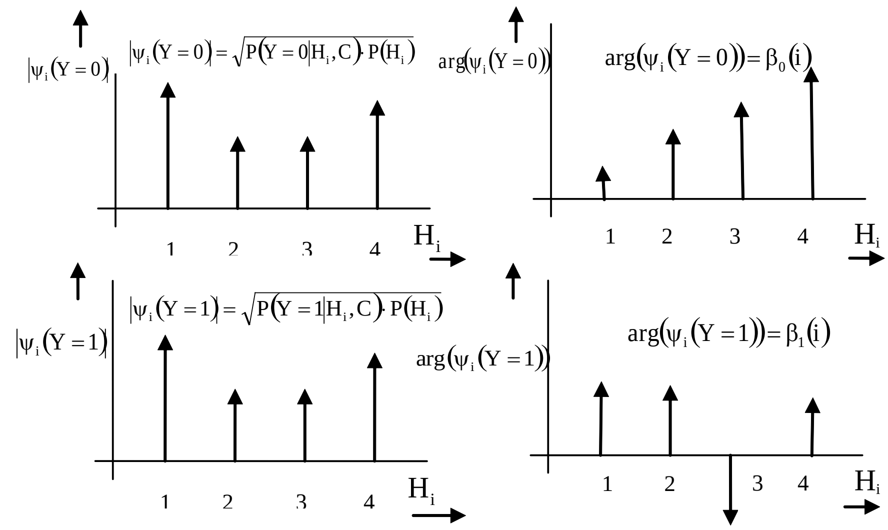


Fig. 8 *Complex multi-models spectrum.*

For better understanding the illustrative example is highlighted. Let two values time series $Y \in \{0, 1\}$ be composed from a mixture of three models described by probabilities $P(Y | H_1)$, $P(Y | H_2)$, and $P(Y | H_3)$, where each component occurs with probabilities $P(H_1)$, $P(H_2)$ and $P(H_3)$. The probabilities $P(Y|H_i)$, $i \in \{1,2,3\}$ are defined in Tab. I and probabilities $P(H_i)$ were chosen:

$$P(H_1) = P(H_2) = P(H_3) = \frac{1}{3}. \tag{59}$$

Model Identification H_i	H_1	H_2	H_3
$P(Y = 1 \mid H_i)$	0.9	0.5	0.4
$P(Y = 0 \mid H_i)$	0.1	0.5	0.6

Tab. I *Real components* $P(Y|H_i), i \in \{1,2,3\}$.

The designer's decision (universal models conditioned by letter C) is given in Tab. II.

Model Identification H_i	H_1	H_2	H_3
$P(Y = 1 \mid H_i, C)$	0.8	0.6	0.7
$P(Y = 0 \mid H_i, C)$	0.2	0.4	0.3

Tab. II *Designer's decision of components* $P(Y|H_i, C), i \in \{1,2,3\}$.

By using the Eqs. (56–58) together with the algorithm [18] the following complex components can be numerically calculated:

$$\psi_1(Y=1) = \sqrt{P(Y=1|H_1,C) \cdot P(H_1)} \cdot \mathrm{e}^{j \cdot \beta_1(1)} = 0.5164, \tag{60}$$

$$\psi_2(Y=1) = \sqrt{P(Y=1|H_2,C) \cdot P(H_2)} \cdot \mathrm{e}^{j \cdot \beta_1(2)} = 0.4472 \cdot \mathrm{e}^{j \cdot 0.5166}, \tag{61}$$

$$\psi_3(Y=1) = \sqrt{P(Y=1|H_3,C) \cdot P(H_3)} \cdot \mathrm{e}^{j \cdot \beta_1(3)} = 0.4830 \cdot \mathrm{e}^{j \cdot 2.4012}, \tag{62}$$

$$\psi_1(Y=0) = \sqrt{P(Y=0|H_1,C) \cdot P(H_1)} \cdot \mathrm{e}^{j \cdot \beta_0(1)} = 0.2582, \tag{63}$$

$$\psi_2(Y=0) = \sqrt{P(Y=0|H_2,C) \cdot P(H_2)} \cdot \mathrm{e}^{j \cdot \beta_0(2)} = 0.3651 \cdot \mathrm{e}^{j \cdot 1.2371}, \tag{64}$$

$$\psi_3(Y=0) = \sqrt{P(Y=0|H_3,C) \cdot P(H_3)} \cdot \mathrm{e}^{j \cdot \beta_0(3)} = 0.3162 \cdot \mathrm{e}^{j \cdot 2.1924}. \tag{65}$$

Based on the Eq. (57) the two complex parameters can be computed as follows:

$$\psi(Y=1) = \psi_1(Y=1) + \psi_2(Y=1) + \psi_3(Y=1) = 0.7746 \cdot \mathrm{e}^{j \cdot 0.7837}, \tag{66}$$

$$\psi(Y=0) = \psi_1(Y=0) + \psi_2(Y=0) + \psi_3(Y=0) = 0.6325 \cdot \mathrm{e}^{j \cdot 1.2596}, \tag{67}$$

where probabilities of falling one or zero could be calculated as follows:

$$P(Y=1) = |\psi(Y=1)|^2 = 0.6, \tag{68}$$

$$P(Y=0) = |\psi(Y=0)|^2 = 0.4. \tag{69}$$

The outcomes (68), (69) are in agreement with the result achieved by the knowledge of model components given in Tab. I and by using Bayes' formula:

$$\begin{aligned} P(Y=1) = P\left(Y=1 \mid H_1\right) \cdot P\left(H_1\right) + P\left(Y=1 \mid H_2\right) \cdot P\left(H_2\right) + \\ + P\left(Y=1 \mid H_3\right) \cdot P\left(H_3\right) = (0.9+0.5+0.4) \cdot \frac{1}{3} = 0.6\,. \end{aligned} \tag{70}$$

The above mentioned numerical example shows that the theory of multi-models composition is feasible. In practical analysis the amplitudes and phases of model components will be estimated from real time series.

5. Conclusion

In this paper the wave probabilistic models were introduced and the mathematical comparison between usually used probabilistic models and wave probabilistic models was presented. With the help of mathematical theory we derived the features of wave probabilistic models. The quantum entanglement or quantization was explained as the consequence of the phase parameters and it can be interpreted as the resonance principle of wave functions. The mathematical theory points out on the applicability of wave probabilistic models and their special features in the area of complex systems.

From the examples given above, we can see the possibility for linking the physical world with the world of information, because every information flow must have its transmission medium, which is typically a physical object (e.g. physical particles) or a certain property of such an object [17]. The case is again similar to an information content, which also must be encoded through a real, physical system. The operations defined above the information systems can then likewise be depicted in a concrete physical environment. Such approach yields to finding better knowledge in the area of information physics [28].

I believe that the capturing of processes in the world around us with the help of information and knowledge subsystems organized into various interconnections (modeled by wave probabilities), especially with feedbacks, can lead to the controlled dissemination of macroscopic work as described by Stuart Kauffman [6], and after the overcoming of certain difficulties, even to the description of the behavior of living organisms or our brain [31].

It seems to be true that the future will bring a convergence of the physical sciences, life sciences and engineering. I would even allow myself to go a bit further, to consider even convergence with the humanities, because I am convinced that the laws of behavior of human society described, for example, in sociology or political science will gain the capacity of being better understood when using the tools of information physics. Wave probabilistic approach can capture Soft Systems Models (SSM) which can bring new quality of understanding the complex systems. Such approach can enrich our learning and we can then speak about quantum cybernetics [21] or quantum system theory [26].

Systemic knowledge is a basis of telematics which is a result of convergence and following progressive synthesis of telecommunication technology and informatics [25]. The effects of telematics are based on synergism of both disciplines. Telematics can be found in a wide spectrum of user areas [33, 40], from an individual multimedia communication towards an intelligent use and management of large-scale networks (e.g. transport, telecommunications, and public service). Advanced telematics provides intelligent environment for knowledge society establishment and allows expert knowledge description of complex systems. It also includes legal, organizational, implementation and human aspects. Transport telematics connects information and telecommunication technologies with transport engineering to achieve better management of transport, travel and forwarding processes by using the existing transport infrastructure.

The telematics can be extended into more complex areas, for example, smart cities or smart regions [30]. Interoperability and cooperation are essential characteristics which a lot of heterogeneous subsystems must posses in order to be integrated [35]. It is understandable that the concept of smart cities/regions yields to an integration of different networks (energy, water, transport, waste, etc.) where the integrated networks must undergo the synergy among the different network sectors to fulfill predefined performance parameters [34]. Designed control system across several sectors form integrated smart networks as a by-product of this approach. Smart approach to complex systems is an example of a multi-disciplinary problem which must – in addition to the technical and technological elements – include the areas of economics, law, sociology, psychology and other humanistic soft disciplines. Only a systemic combination of these elements can achieve the goal which is generally expected from the smart systems.

The inspiration for the above defined problems came from quantum physics [4, 5]. The analogy with quantum mechanics could be seen as very interesting and is likely to bring a lot of inspiration for the future work within the complex systems modelling by wave probabilistic functions. The presented results should not be treated as a finished work, but rather the beginning of a journey. It is easy to understand that a lot of the mentioned theoretical approaches should continue to be tested in practical applications.

Curriculum Vitae

Miroslav Svítek was born in Rakovník, Czech Republic, in 1969. He graduated in radioelectronic from Czech Technical University in Prague, in 1992. In 1996, he received the Ph.D. degree in radioelectronic at Faculty of Electrical Engineering, Czech Technical University in Prague. Since 2002, he has been associated professor in engineering informatics at Faculty of Transportation Sciences, Czech Technical University in Prague. Since 2005, he has been nominated as the extraordinary professor in applied informatics at Faculty of Natural Sciences, University of Matej Bel in Banska Bystrica, Slovak Republic. Since 2008, he has been full professor in engineering informatics at Faculty of Transportation Sciences, Czech Technical University in Prague. He is currently teaching courses and doing research in theoretical telematics, intelligent transport systems, smart cities, quantum system theory and quantum informatics. Miroslav Svítek is president of Association of transport telematics of the Czech and Slovak Republic (it covers more than 80 public and private organizations), member of Engineering academy of the Czech Republic and Dean of Faculty of Transportation Sciences, Czech Technical University in Prague. He is author or co-author of more than 250 scientific papers and 10 monographs.

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