MODELLING AND OPTIMIZATION OF AN INTELLIGENT ENVIRONMENTAL ENERGY SYSTEM IN AN INTELLIGENT AREA

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Abstract: The article deals with the current state of energy consumption, the development of distribution networks in the context of its decentralization and integrated community energy systems. The article focuses on the issue and optimization of the operation of EnergyHubs (EH) – energy centres in terms of solving environmental aspects using a mathematical model in the GAMS environment. The acquired knowledge and results of simulations were then applied to a specific urban area to find the optimal variant of EH. The aim of the research is to present its results at the level of cleaner production, improvement of the environment, significant reduction of CO^2 and sustainability of society. My experience proves that the achievement of sustainable development goals represents fundamental gaps in research and practical applications, especially at the level of specific projects. It is mainly the application of insufficient indicators and work methodologies in the design of building projects with almost zero energy consumption. Another shortcoming is the coordination of design procedures and applications of optimization and simulation methods necessary to address the energy performance of buildings or clusters of buildings. In addition, the results show growing expectations about the added value of applying artificial intelligence in meeting sustainable development goals, through new data sources that inevitably enter the energy sustainability design process.

Key words: *energy consumption, passive distribution networks, active distribution networks, integrated community energy system, EnergyHub (EH), energy centre, mathematic model, environmental aspects, GAMS (General Algebraic Modelling System)*

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1. Introduction

With increasing energy consumption and efforts to reduce greenhouse gas emissions, the development of renewable energy resources and the reduction of the use of fossil fuels by the European Union, it is necessary to find solutions for energy

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consumption reduction. Human activity affects the world we live in, and the ability to sustain life is crucial to our future. Environmental science can help to quantify this impact, understand how science affects policy, and how policies affect the environment. Aspects that can significantly affect the environment should not be isolated activities, but they should be part of the organization's projects and processes. It is necessary to use the ČSN EN ISO 14031 – *Evaluation of Environmental Performance* standard and also ČSN EN ISO 14004 – *Environmental Management Systems – General Guidelines* standard for implementation to determine suitable indicators. The indicators can include, e.g.: electricity consumption, fuel consumption (petrol, diesel, LPG, CNG), tap water consumption, surface water and groundwater consumption, paper consumption, heat consumption for heating, etc. Decentralized energy is a term that is frequently used in promotional materials of foreign and, of course, Czech environmental movements. Applied to electricity, it is – very generally speaking – a transition from the current energy concept based on a limited number of production sources with high installed capacity to a large number of local renewable energy sources (RES) with significantly lower installed capacity, which would produce a comparable amount of electricity, thanks to their a huge number.

This is a process that significantly supports the importance of the meaning of environmental policy in science and its applications. A significant share of small renewable sources enables decentralized energy to significantly reduce emissions of pollutants and greenhouse gases from the assessed options. The application of renewable energy sources (RES) creates unprecedented challenges for regional energy systems to maintain the flexibility and reliability of the system.

Power system operators may limit some of the renewable energy produced at different intervals. From an economic-environmental point of view, the development of sustainable energy must focus on creating a system of:

- Renewable energy sources according to acceptable conditions in the area,
- Application of the best available techniques in the implementation of new sources, so that they include conditions of economic acceptability,
- Energy saving measures in the field of final consumption and primary energy sources with the development of a detailed methodology,
- National action plan for savings (energy efficiency),
- A legislative framework for ensuring sustainable energy development planning with regard to "Environmental Policy" and "Climate Protection Policy".

In this article, I will focus on the above aspects in the context of a proposal based on my research in the field of project creation of the decentralized energy system and thus EH [3].

One of these options is the development of integrated community energy systems and their efficient connection to energy distribution networks. One of the options of effective connection of these community systems to distribution networks is the use of EH or energy centres.

The aim is to analyse the current state of energy consumption, present the available options for the development of distribution systems, use decentralized energy sources, summarize the benefits of integrated community energy systems, and especially to introduce EH [4] for possible reduction of energy consumption, efficient use of energy resources and benefits associated with the possibility of the energy conversion between the carriers of these energies.

A significant part of this work, presented briefly in this article, also deals with the mathematical model of EH in the GAMS program, which allows finding of the optimal variant and behaviour of EH over time, using linear programming. Grid-connected are seen as a way forward in order to increase system flexibility, reduce renewable energy limitations, and increase energy efficiency. In practice, simplified models can significantly affect the performance of grid-connected EH. Therefore, this article proposes a holistic structure to determine the optimal coordinated operation of grid-connected EH and regional power system by relying on high-performance photovoltaic systems. The level of emissions and the amount of renewable energy applied, together with the total operating costs of the integrated energy system of EH, are among the main objectives of the optimization issue.

The findings are applied in the practical part of this article, where a mathematical model in the GAMS program is used to find the optimal variant of EH for the given area of M. J. Lermontova – Ve Struhách, Prague 6, Czech Republic.

2. General model of an Energy Hub

The future visions of energy networks, including several energy carriers, as multi-carrier systems and hybrid systems, allows for greater flexibility in integrated network operation as well as the necessary energy optimization. In fact, different infrastructures can influence each other in terms of energy flow, including their storage, etc. In this case, the so-called EH play a crucial role in the connection points between different infrastructures enabling the flow of energy through different networks. The combination of several converters in these hubs is a necessary motivation for the integration of multiple energy carriers. From this point of view, different support structures can be used by different carriers to provide different forms of energy at the output port. I evaluate the optimization of the energy flow based on the need, e.g. energy consumption costs, emission costs, energy losses, etc., as the basic process of designing a Smart Area environmental system. The state of the system in terms of all control and state variables, including energy flows, is defined by other variables.

I present the EH concept and its modelling, including the optimization of the hybrid electricity system and gas network. The general framework for modelling power systems based on the hub concept is little known at this time. It is a medium-term management of EH based on the price of electricity and the uncertainty of solar radiation, which was documented within the application in the EH.

The general model of the EH, presented by Göran Andersson from the Swiss Federal Institute of Technology, can be imagined as seen in Fig. 1. This general model was created for the load, induced especially by a metropolitan area [5].

Decentralized energy source (DES), see Fig. 1, is understood as such a source that is located close to the final consumer and is most often connected to the distribution network of the electricity system. Most of them are small to medium power sources producing power in kilowatts to tens of megawatts. The benefits of

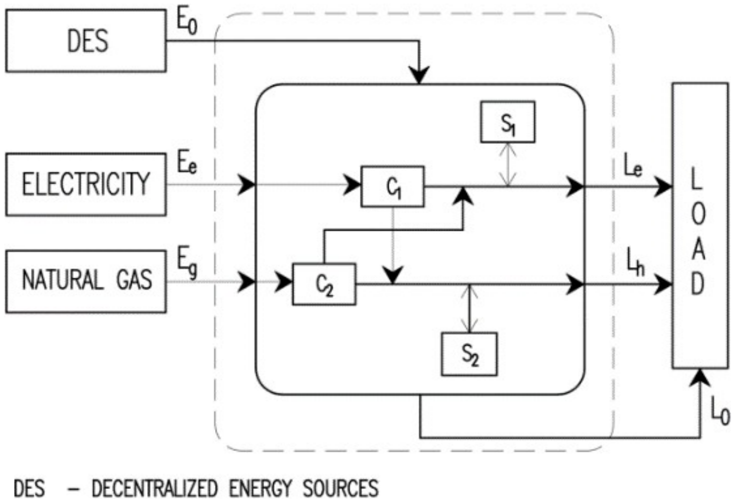


Fig. 1 *General model of an EnergyHub [5].*

decentralized electricity sources are both economic and environmental. In the case of cogeneration units, it is mainly their high efficiency, which can climb up to 95 %, which is directly reflected in the savings of primary fuel. For renewable energy sources, their environmental benefits are generally apparent. Decentralized sources also contribute to the safety and reliability of the electricity system. Recently, these smaller sources have been also used in the planning of so-called Smart Grids.

An EH can be considered an object which can convert, regulate and store multiple energy carriers and which represents the interface between different energy infrastructures. In a multi-energy hub, also referred to as EH, various commands are generated within the power flow that meet the output power requirements. A simplified structural diagram of the energy in the EnergyHub is shown in Fig. 1. In Fig. 1, E_0, E_e, E_g represent energy from renewable energy sources (RES), electricity from distribution and natural gas at the input to the EH, L_e a L_h represent the electrical and thermal power of the EH and L_0 represents the electricity from the RES. On the supply side of the EH, there are electricity, natural gas, and heat, and at the demand side of the user includes the demand for electricity, heat and cold. A coupling matrix model is used to create input carriers that are mapped to the outputs of the EH, as shown in Fig. 3. [10] Furthermore, the conversions (c_1, c_2) and storages (s_1, s_2) of these energies are included in the model. The basic principle of energy input, conversion, and output by EH can be mathematically expressed by a coupling matrix and graphically represented as seen in (Fig. 2, Fig. 3).

The matrix model in Fig. 3 is used to create input carriers that are mapped to the outputs of the EH, where in Fig. 3 c_{ij} is the conversion factor between input and output energy forms, where $0 \leq c_{ij} \leq 1$. $E(1, \ldots, m)$ is the input energy (input power) and $L(1, \ldots, n)$ is the output energy (transferred power) from the EH [5]. The EH model is mathematically described in the matrix – vector in Fig. 3.

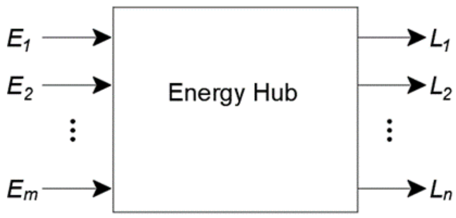


Fig. 2 *Inputs and outputs of an EnergyHub [5].*

$$\begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nm} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_m \end{bmatrix}$$

Fig. 3 *Coupling matrix of an EnergyHub [5].*

$$[E'] = [\nu] \times [N] \times [L]\,, \tag{1}$$

$$[L] = [\mu] \times [N] \times [E]\,. \tag{2}$$

The energies at the EH input are defined as E' see (1) and the output vector of the electricity carrier is defined as L. ν is a planning factor whose range is $0 \leq \nu \leq 1$. μ and N are matrices of efficiency of the EH factor schedule.

For subsequent comparison of the EH model with existing traditional energy supply models, the following Fig. 4. Can be used:

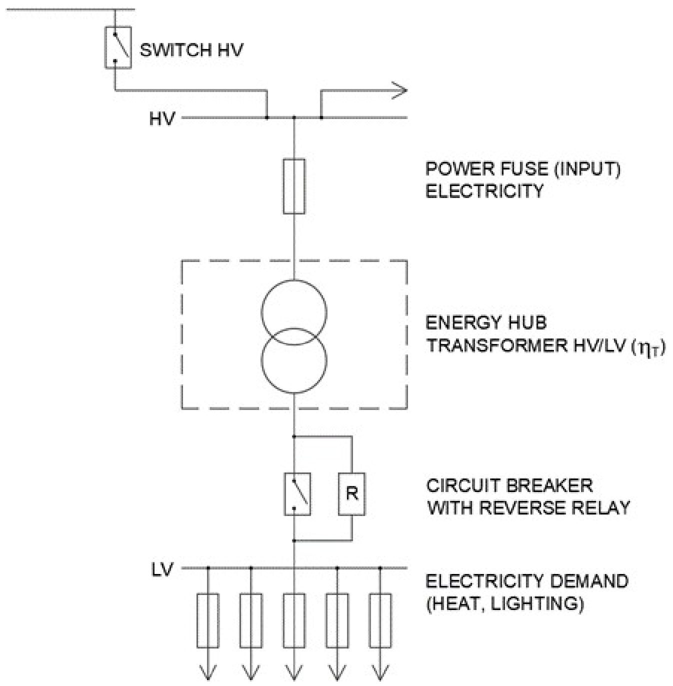
Fig. 4(b) shows the supply of electricity only. It is used both to satisfy the demand for heat and for other common consumption such as lighting. Fig. 4(a) shows the supply of both electricity and gas, e.g. supply in areas of suburbs of larger cities, for example, where electricity is used for common consumption and where heat demand is satisfied by burning gas in a gas boiler. Scheme (a) can be mathematically expressed as follows:

$$E_e^{in}(t) = L_e/\eta_T, \tag{3}$$

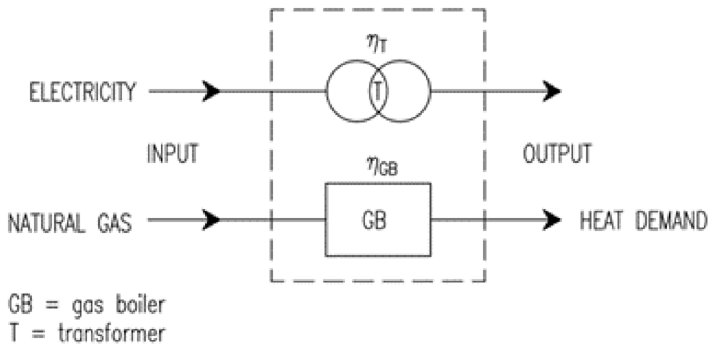
$$E_g^{in}(t) = L_g/\eta_{PK}, \tag{4}$$

where E_e^{in}, E_g^{in} is the energy of electricity and the energy of gas at the input, L_e, L_g is the load of electricity and gas at the outlet and η is the conversion efficiency. For comparison with traditional energy supply models, the structure of EH can be imagined as shown in Fig. 5.

In EH, various forms of energy are received on input ports connected to the energy infrastructure and energy services in the form of electricity, heating and cooling are delivered on output ports [6]. Within EH, various forms of energy are



(a)



(b)

Fig. 4 *Traditional models.*

converted and treated using *converter technologies* such as, Fig. 5: *transformers, alternating energy, CHP technology, heat exchangers and absorption chillers.* The demand for each type of energy varies considerably during the day and therefore, it is random. Besides, in a competitive electricity market, electricity prices often change over time, thus optimizing.

For one EH input with one output, the coupling factor is the efficiency of the μ converter. When considering multiple inputs, so-called multi-output systems, the coupling factor is generally represented as a matrix corresponding to the efficiency of the converter. A planning factor is introduced that determines how the energy at the EH input is distributed through the converter (conversion), Fig. 5. For simplicity, it is assumed that the efficiency of the transducer is constant, which leads to a constant efficiency of the matrix. In this article (study and subsequent implementation), the planning factor is a variable factor in the operation of a multi-energy hub system. The EH optimizes the planning relationship between the input and output energy by adjusting the planning factor.

The EH in Fig. 1 is considered as an example, but energy storage and cooling are not considered. The inputs of the EH are electricity and natural gas and the outputs are electrical load and heat load. Using the law of conservation of energy, the output of the converter is expressed as the product of input and efficiency.

The energy carriers received at the input port of the charge converters are represented as L_i $(i = 1, 2, \ldots, n)$ and the input vector is defined as E_i $(i = 1, 2, \ldots, m)$. v is the planning factor, the range of which is 0 or 1 and N are the matrices of efficiency and the schedule of EH factors.

$$L_e\left(t\right) - \mu_{Conv} \cdot P_w\left(t\right) = L_e^{net}(t), \tag{5}$$

$$\left.\begin{aligned} L_e^{net}\left(t\right) &= \mu_{ee}^{T} P_e\left(t\right) + \nu \mu_{ge}^{GT} P_g\left(t\right) \\ L_h\left(t\right) &= \left[\nu \mu_{gh}^{GT} + (1-\nu)\mu_{gh}^{F}\right] \cdot P_g(t). \end{aligned}\right\} \tag{6}$$

After setting the relational matrix between the input values and the output load in each EH, the number of input values can be obtained by the following mathematical expression:

$$\begin{bmatrix} L_e^{net}(t) \\ L_h(t) \end{bmatrix} - [N_t] \times \begin{bmatrix} P_e\left(t\right) \\ P_g\left(t\right) \end{bmatrix} = 0, \tag{7}$$

$$\begin{bmatrix} P_e(t) \\ P_g(t) \end{bmatrix} = \left([N_t] \times [\mu]\right)^{-1} \begin{bmatrix} L_e^{net}(t) \\ L_h(t) \end{bmatrix}. \tag{8}$$

In equations (5)–(7) $P_e(t)$ and $P_g(t)$ represent the input power of natural gas to the EH at time t; $L_e(t)$ and $L_h(t)$ represent the user's energy and heat consumption at the given time; $P_w(t)$ represents the photovoltaic energy input at time t; $L_e^{net}(t)$ represents the demand for energy that the user needs from EH; $\mu_{ee}^{T}, \mu_{ge}^{GT}, \mu_{gh}^{GT}$, and μ_{gh}^{F} are the conversion efficiencies of the transformer, gas, and electricity in the eventual connection of cogeneration (CHP), as well as the gas for the heat demand in the CHP and the gas boiler in the EH. $P_w\left(t\right)$ represents the strength of the photovoltaic energy at the time of the curve t, which depends on the characteristics of the solar radiation.

Fig. 5 illustrates the different levels of general EH. In EH, various forms of energy are received at the input ports connected to the energy infrastructure and energy services in the form of electricity, heating and cooling are supplied to the output ports [6].

The main advantage of EH is their efficient use of multi-generation (cogeneration, three-generation, poly-generation) systems in order to optimally use energy

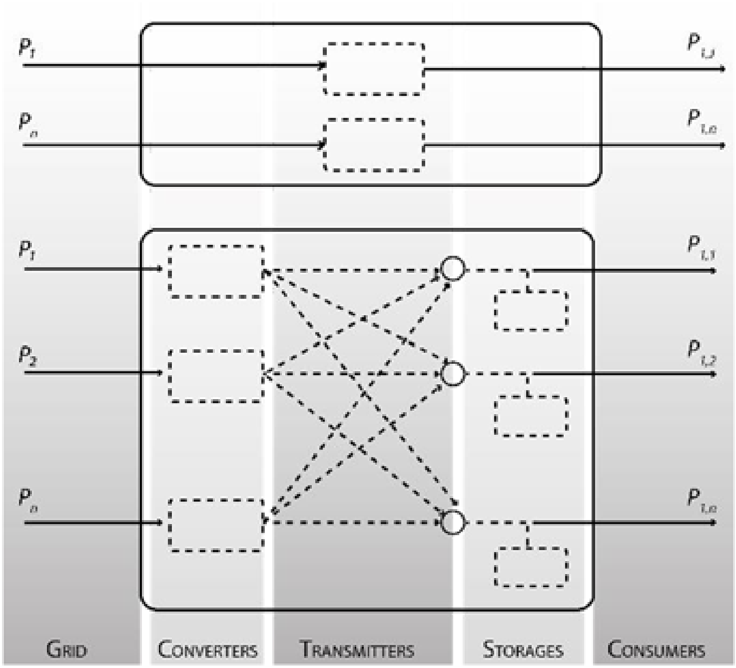


Fig. 5 *General structure of an EnergyHub [8].*

sources, increase efficiency, and reduce costs and emissions. Although these technological structures are widely used in the available literature, their unambiguous definition has not yet been established in the case of EH. According to academics from the University of Tehran (the capital of Iran), these systems can be called integrated EH and their definition is as follows: Integrated EH are multigenerational systems in which different energy carriers are produced, transmitted, stored and consumed to meet different types of demand [3].

EH have four main functions: energy input, energy conversion (production), energy storage, and energy output. Energy input is represented by: natural gas, liquid fuels (diesel, petrol), wind energy, solar energy, biomass, geothermal energy, energy distribution networks, and industrial waste heat. The conversion components are: transformers, heat exchangers, sorption coolers, electric heaters, and heat pumps.

The Y-type matrix is used to compare three groups of data simultaneously. On the one hand, the matrix diagrams show the interrelationships between the individual items and at the same time demonstrate the possible mutual independence of one item from the others.

Equations and inequalities in the input file are reflected in the output file. Here, these equations are written in the form that GAMS uses for its own calculation – unknown on the left, constants on the right.

3. Mathematical model of the EnergyHub

The mathematical model of the EH is given by the objective evaluation function compiled according to [3], the limitations given by the EH components and the balance of energy flows through the EnergyHub.

3.1 Objective function

The objective evaluation function of the EH mathematical model is based on the prices of input commodities and the daily consumption of these commodities. The criterion for this evaluation function is therefore the sum of the costs of these commodities [3, 11].

$$\min OF = \sum_t \lambda_t^a A_t + \lambda_t^b B_t + \cdots + \lambda_t^n N_t, \tag{9}$$

where $\lambda_t^a, \ldots, \lambda_t^n$ unit prices of input commodities and $A_t, \ldots, N_t$ are consumptions of these commodities.

3.2 Limitations and energy balances

The amounts of energies, which enter the EH and which are needed to meet the demand through the conversion, production, or storage of these energies by the EH component are given by the sum of the energies entering the EH components [3, 11].

$$P(t) = \sum_t A_{i,t} + A_{j,t} + \cdots + A_{n,t}, \tag{10}$$

where $A_{i,t}, \ldots, A_{n,t}$ is the amount of input energies entering the EH. The amounts of energies at the output of the EH, which are needed in order to meet the demand, are given by the sum of the energies at the output from the EH components [4].

$$L(t) = \sum_t B_{i,t} + B_{j,t} + \cdots + B_{n,t}, \tag{11}$$

where $B_{i,t}, \ldots, B_{n,t}$ is the amount of energies at the output of the EH.

3.3 Limitations given by the energy supplies by the networks

The amount of energies entering the EH from distribution networks is limited by the connection capacity of these distribution networks. Alternatively, this limitation is contractually agreed [3].

$$P_x^{\text{net}}(t) \leq P_x^{\max}, \tag{12}$$

where P_x^{net} is the immediate amount needed to operate the EH and $P_x^{\max}$ is the capacity of the connection lines [3].

3.4 Limitation of the conversion and production

The conversion of energies from one form to another, its production or possibly the conversion of energy to its lower level is not lossless. At the same time, the components that enable the conversion or production of energy operate within their operating ranges. Conversion, conversion efficiency, and operating ranges are expressed as follows [3].

$$B_t^{\min} < B_t < B_t^{\max}, \tag{13}$$

$$B_t = \eta_x A_t, \tag{14}$$

$$\eta_x < 1, \tag{15}$$

where $B_t^{\min}$ is the minimum operating performance of the component, B_t is the immediate operating performance of the component and $B_t^{\max}$ is the maximum operation performance of the component. Functioning of the components can be expressed as the production of energies B_t by the consumption of the input commodity A_t with the given efficiency η_x. The primary efficiency η_x is always lower 1.

3.5 Energy storage limitation

The EH can have three possible types of storage – storage of cold, heat, and electricity. However, in principle, all of these types of storage work in the same way, which can be expressed by the following equations [9].

$$SOC_t = SOC_{t-1} + \left(A_t \eta_a - \frac{B_t}{\eta_b} \right) \Delta_t, \tag{16}$$

$$A_{\min} \le A_t \le A_{\max}, \tag{17}$$

$$B_{\min} \le B_t \le B_{\max}, \tag{18}$$

$$SOC_{\min} \le SOC_t \le SOC_{\max}. \tag{19}$$

The SOC variable expresses the state of charge of the storage, A is the amount of energy entering the storage and B represents the amount of energy at the output of the storage. The first equation shows the change in charge level from the previous state with charging efficiency η_a or discharging efficiency η_b. The limits for charging or discharging the storage are limited by maximum and minimum values, i.e. $A_{\min}$, $A_{\max}$, $B_{\max}$, $B_{\max}$, and the storage charge level is defined by the minimum storage capacity value $SOC_{\min}$ and the maximum storage capacity value $SOC_{\max}$.

3.6 Price of energies (rates)

The price of energies plays a critical role in the objective evaluation function. It can be constant over time, as in the case of gas prices, or determined by dynamic methods. When determining the price of energies, they are used e.g. for electricity [4].

3.7 Optimization

Optimization to solve practical problems is very widespread in scientific research today and its use has a growing tendency in the field of energies. Recent research on optimizing the EnergyHub operation offers a variety of objective evaluation functions and limitations and it uses various solution tools, including higher-level programming languages such as Delphi, MATLAB, or Fortran. A very widespread higher-level programming language, which has recently been widely used to optimize EH, is GAMS, which uses its built-in algorithms (solvers) that solve the issue of optimal operation of EH.

EnergyHub was investigated from the point of view of a mathematical model and its optimal functioning. This sub-goal was achieved using GAMS software, which allows you to compile, analyze and solve complex mathematical models [6].

The composition of the EH, i.e. its inputs and components, has a great impact on its functioning as a whole. By selecting suitable components and energy resources, it is possible to find the most suitable and optimal variant of the EH to minimize the objective evaluation function based on the given input data of energy demand of the consumers. The aim of EH optimization is to find the most advantageous combination of energy inputs and components of production, conversion, and storage in the EH to minimize e.g. operating costs or CO^2 emissions [3, 4].

4. General Algebraic Modelling System GAMS

The general algebraic modelling system is a high-level modelling system for mathematical programming and optimization. It is designed for complex large-scale modelling applications and it allows creation of large, sustainable models that can be quickly adapted to new situations. GAMS allow formulating of mathematical models in a form very close to their mathematical description, but at the same time, the language of the GAMS system is similar to common programming languages. Therefore, it allows mathematical models to be understood and maintained not only by programmers but also by scientists in the field. For a detailed description of the functioning of the mathematical model, a complex variant of the composition of components was chosen.

Fig. 6 shows the component layout of the default EH model [9].

4.1 Components of the default model of an EnergyHub and mathematical description of their operation

Battery storage (BS) allows storing of electricity and its subsequent use at the time when it is most needed. This may be the time when there is peak consumption in the network and the immediate consumption of end customers' needs to be covered, or it may also be the time when the hourly rate for electricity is high and the battery storage is sufficiently charged. In this case, the electricity stored in the BS can be used to reduce the consumption of expensive electricity from the distribution network at this time and thus save money associated with its consumption. The BS makes it possible to reduce the costs associated with the consumption of electricity from the distribution network by being able to charge at

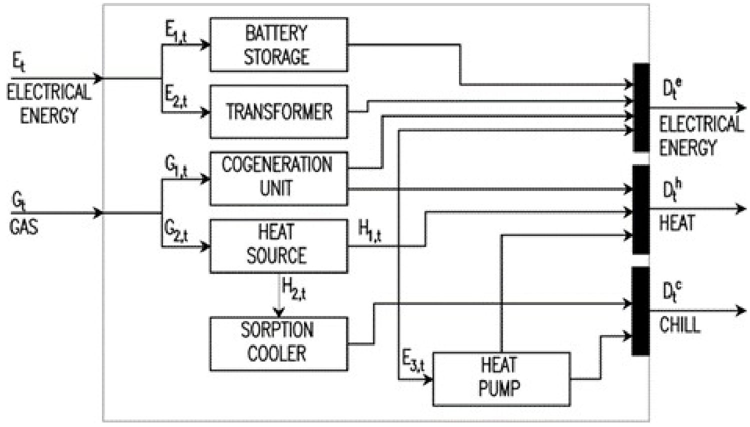


Fig. 6 *Default model of an EnergyHub [9].*

a time when electricity is cheap and then provide such cheaply charged electricity to customers at a time when direct consumption of electricity from the distribution network would be expensive. The mathematical model of the BS can be expressed as follows [9].

$$SOC_t = SOC_{t-1} + \left(E_t^{ch} \eta_c - \frac{E_t^{dch}}{\eta_d} \right) \Delta_t, \tag{20}$$

$$E_{\min}^{ch} \leq E_t^{ch} \leq E_{\max}^{ch}, \tag{21}$$

$$E_{\min}^{dch} \leq E_t^{dch} \leq E_{\max}^{dch}, \tag{22}$$

$$SOC_{\min} \leq SOC_t \leq SOC_{\max}, \tag{23}$$

$$I_t^{dch} + I_t^{ch} \leq 1, \tag{24}$$

$$I_t^{ch} I_t^{dch} \in \{0, 1\}, \tag{25}$$

where SOC_t is the BS state in the given hour, $E_t^{ch/dch}$ expresses the amount of charged or discharged electricity at the given hour and the I_t^{ch}, I_t^{dch} variables express whether the BS is charging or discharging at the given hour. The η_c and η_d variables express the efficiency of the charging, or discharging of the BS. The $SOC_{\min}$ and $SOC_{\max}$ variables express maximum or minimum capacity of the BS and the $E_{\max}^{ch/dch}$ and $E_{\min}^{ch/dch}$ variables express maximum and minimum limits of charging / discharging of the BS.

Transformer For the needs of the mathematical model of the EH, the transformer can be expressed as follows:

$$E_t^{out} = \eta_{ee} E_t^{in}. \tag{26}$$

The conversion of the voltage level is not lossless, therefore it is necessary to introduce the efficiency of the transformer $\eta_{ee} \cdot E_t^{out}$ is the output energy in time and E_t^{in} is the energy at the input of the transformer.

Cogeneration unit CHP uses primary energy as less polluting energy and provides highly efficient electrical and thermal loads. The current model of combined heat and power generation has been strongly developed around the world. The total capacity is expected to reach 483.7 GW by 2023 [12].

The combined production of electricity and heat can be mathematically described as follows:

$$H_t = \eta_{gh}^{chp} G_t, \tag{27}$$

$$E_t = \eta_{ge}^{chp} G_t. \tag{28}$$

The first equation expresses the amount of heat H_t produced by the cogeneration unit by burning gas G_t with efficiency η_{gh}^{chp}. The second equation then expresses the amount of produced electricity E_t, produced by burning the amount of gas G_t with efficiency η_{gh}^{chp}.

At a higher level, a combined cooling, heat and energy (CCHP) model was introduced in [13–15] in order to supplement the additional cooling need of an air conditioning system (AC) or an absorption cooler (ACh).

Boiler The process of thermal energy production by burning gas in the boiler is expressed by the following equation:

$$H_t = \eta_{gh} G_t, \tag{29}$$

where H_t is the amount of thermal energy produced, η_{gh} is the boiler efficiency, and G_t is the amount of fuel.

Sorption cooler Mathematically, the principle of operation of a sorption cooler can be expressed as follows:

$$C_t = \eta_{hc} H_t, \tag{30}$$

where C_t is the amount of produced cold, η_{hc} is the heat to cold conversion efficiency, and H_t is the amount of heat delivered to the cooler.

Heat pump The whole process of operation of the heat pump is mathematically expressed by the following equations:

$$C_t + H_t = E_t \times COP, \tag{31}$$

$$H_t^{\min} I_t^h \leq H_t \leq H_t^{\max} I_t^h, \tag{32}$$

$$C_t^{\min} I_t^c \leq C_t \leq C_t^{\max} I_t^c, \tag{33}$$

$$I_t^c + I_t^h \leq 1, \tag{34}$$

$$I_t^c \times I_t^h \in \{0, 1\}. \tag{35}$$

The heat pump can operate in either cold or heat production mode. These states are expressed by the I_t^c or I_t^h variable. The cooling performance C_t or heating performance H_t is within the range of the minimum cooling performance $C_t^{\min}$ or minimum heating performance $H_t^{\min}$ and the maximum cooling performance $C_t^{\max}$ or maximum heating performance $H_t^{\max}$.

4.2 Equation of the mathematical model of the default EnergyHub

The mathematical model of the Default EH model is composed of the following equations [5, 9].

Objective function

$$\min OF = \sum_t \lambda_t^e E_t + \lambda_t^g G_t. \tag{36}$$

The resulting value is given by the sum of the consumptions of individual input commodities multiplied by the unit price of these commodities. With the procedure chosen in this way, the most optimal solution is sought by minimizing operating costs – i.e. minimizing the amount paid for the input commodities consumed by the EH [3].

The objective function of the EH model does not have to be based on the financial aspect depending on the amount of input commodities consumed, but it can be based on other various aspects such as emissions of CO_2 or other pollutants. In that case, the equation would not include the unit prices of these commodities, but, for example, the amount of CO_2 produced per units of commodities consumed. On the left side of the equation, there is the OF variable, which needs to be minimized. On the right side of the equation, there is the sum of the $\lambda_t^e E_t$ and $\lambda_t^g G_t$ products, where λ_t^e is the electricity price, λ_t^g is the gas price, E_t is the amount of consumed electricity, and G_t is the amount of consumed gas.

Electricity at the EnergyHub input

$$E_t = E_{1,t} + E_{2,t}. \tag{37}$$

This equation expresses the balance of electricity flow at the EH input. The total flow of electricity entering the EH E_t is the sum of the partial flows of electricity entering the battery storage $E_{1,t}$ and the flow of electricity entering the transformer $E_{2,t}$.

Electricity at the EnergyHub output

$$\eta_{ee} E_{2,t} + E_t^{dch} + \eta_{ge} G_{1,t} = D_t^e + E_{3,t}. \tag{38}$$

The first term, $\eta_{ee} E_{2,t}$, expresses the flow of electricity through the transformer. The flow of electricity into the transformer is expressed as $E_{2,t}$ and the transformer efficiency is expressed as η_{ee}. The second term of the left side of the equation, E_t^{dch}, is the electricity flow from the BS; this term is non-zero only at the time when it is suitable to use the electricity stored in the BS. The third term on the left side of the equation, $\eta_{ge} G_{1,t}$, expresses the electricity production by the cogeneration unit. $G_{1,t}$ is the gas consumption and η_{ge} is the efficiency of gas to electricity conversion by cogeneration unit. On the right side of the equation, there are is a sum of two terms that express the electricity required from the EH. The first term of the right sight, D_t^e , stands for the demand for electricity from the customers connected to the EH and $E_{3,t}$ expresses the electricity needed for the heat pump operation.

Input of Electricity to the Battery Storage

$$E_{1,t} = E_t^{ch}. \tag{39}$$

This equation assigns a part of the electricity flow entering the EH $E_{1,t}$ as the electricity flow to the BS E_t^{ch}.

Status and change of the BS charge

$$SOC_t = SOC_{t-1} + \left(E_t^{ch}\eta_c - \frac{E_t^{dch}}{\eta_d}\right)\Delta t. \tag{40}$$

SOC_t stands for the current state of charge of the BS at the given time and is given by the sum of the terms on the right side of the equation. The right side of the equation expresses the change in the state of charge of the BS. The SOC_{t-1} term stands for the state of charge of the BS at the previous time. The terms in parentheses describe the process of charging or discharging the BS. The BS charging process is described by the $E_t^{ch}\eta_c$, term, which expresses the amount of electrical energy that charges the BS. It consists of the E_t^{ch}, variable, which is the flow of electricity to the BS, and which is adjusted (multiplied in this case) by the efficiency of the BS charging process, η_c. The discharging process is defined by the negative $\frac{E_t^{dch}}{\eta_d}$, term, which expresses the amount of electricity, which discharges th BS. The E_t^{dch} variable is the flow of electricity from the BS, which is adjusted (in this case divided) by the efficiency of the BS discharging process, η_d.

BS operating mode

$$I_t^{dch} + I_t^{ch} \leq 1, \tag{41}$$

$$I_t^{ch} I_t^{dch} \in \{0, 1\}. \tag{42}$$

BS has two operating modes – charging or discharging of BS batteries. These modes cannot run at the same time, so the binary variables I_t^{ch} and I_t^{dch}, are used here. They determine in which mode the BS is currently operating. When the BS is charging, the I_t^{ch} value is 1; when the BS is not charging, I_t^{ch} acquires the value of 0. The same applies to discharging of the BS. If the BS is discharging, the I_t^{dch} variable acquires the value of 1, and when the BS is not discharging, the I_t^{dch} variable acquires the value of 0. This constraint is expressed by the first equation of the above pair, which states that the sum of these variables must be less than or equal to 1. Since these are binary variables and their value can be only 1 or 0, there can be no situation where the BS would be both charging and discharging.

Charging Range of the BS

$$E_{\min}^{ch} I_t^{ch} \leq E_t^{ch} \leq E_{\max}^{ch} I_t^{ch}, \tag{43}$$

$$E_{\min}^{ch} I_t^{ch} \leq E_t^{ch} \leq E_{\max}^{ch} I_t^{ch}. \tag{44}$$

BS charging is limited by the minimum and maximum value. BS charging, E_t^{ch} is limited by the minimum value, $E_{\min}^{ch}$, and by the maximum value, $E_{\max}^{ch}$. The I_t^{ch}, term, which acquires either the value of 1 or the value of 0, determines whether or not the BS is charging at the given moment.

Discharging Range of the BS

$$E_{\min}^{dch} I_t^{dch} \leq E_t^{dch} \leq E_{\max}^{dch} I_t^{dch}. \quad (45)$$

BS capacity

$$SOC_{\min} \leq SOC_t \leq SOC_{\max}. \quad (46)$$

$SOC_{\max}$ is the maximum capacity of the BS, $SOC_{\min}$ specifies the lower limit for battery discharge. The immediate state of charge of the BS batteries, SOC_t, is between these values, as shown by this equation:

Gas at the EnergyHub input

$$G_t = G_{1,t} + G_{2,t}. \quad (47)$$

The gas entering the EH, G_t is the sum of the gas flowing into the cogeneration unit, $G_{1,t}$ and the gas flowing to the gas boiler $G_{2,t}$.

Heat at the EnergyHub input

$$\eta_{gh} G_{1,t} + H_{1,t} + H_t^{EHP} = D_t^h. \quad (48)$$

D_t^h is the output of thermal energy, $\eta_{gh} G_{1,t}$ expresses the output of thermal energy from the cogeneration unit, where the η_{gh} variable is the efficiency of conversion of gas to thermal energy and $G_{1,t}$ is the amount of gas entering the cogeneration unit. The $H_{1,t}$ term is the amount of thermal energy coming out of the gas boiler intended directly to satisfy the demand for thermal energy and the H_t^{EHP} term is the amount of thermal energy coming out of the heat pump.

Gas boiler

$$\eta_{gh}^f G_{2,t} = H_{1,t} + H_{2,t}. \quad (49)$$

The amount of gas, $G_{2,t}$, is brought to the input of the gas boiler and burned with the efficiency η_{gh}^f producing thermal energy $H_{1,t}$, which is the consumer's demand for thermal energy and $H_{2,t}$ is the thermal energy intended for the production of cold by the sorption cooler.

Cold at the EnergyHub output

$$\eta_{hc} H_{2,t} + C_t^{EHP} = D_t^c. \quad (50)$$

D_t^c is the consumer demand for cold from the EH, $\eta_{hc} H_{2,t}$ is the cold output from the sorption cooler. The amount of thermal energy entering the sorption cooler, $H_{2,t}$, is converted to cold with the heat-to-cold conversion efficiency of the sorption cooler, η_{hc}. The C_t^{EHP} term of the equation expresses the cold output from the heat pump.

Heat pump operating mode

$$I_t^c + I_t^h \leq 1, \tag{51}$$

$$I_t^c I_t^h \in \{0, 1\}. \tag{52}$$

The mode of heat or cold production is expressed by binary variables I_t^c and I_t^h. These variables acquire values of 1 or 0, depending on the mode in which the heat pump is operating. To express the operating mode of the heat pump, the first equation of the above two is introduced into the model, which guarantees that the heat pump always operates in only one of these modes at the time, because at the given time, the sum of the binary variables I_t^c and I_t^h can only be smaller equal to 1.

Heat pump performance

$$C_t^{EHP} + H_t^{EHP} = E_{3,t} \times COP. \tag{53}$$

The amount of produced heat, H_t^{EHP}, or cold, C_t^{EHP} produced by the heat pump is directly proportional to the consumed electricity entering the heat pump, $E_{3,t}$ adjusted by the efficiency of the heat pump, COP, which is the heating factor of the heat pump (Coefficient of Performance).

Heat pump heating performance

$$H_t^{\min} I_t^h \leq H_t^{EHP} \leq H_t^{\max} I_t^h. \tag{54}$$

The maximum amount of heat produced by the heat pump is limited by its performance. Limitation of the minimum heat production applies. To apply this limitation, the value of the H_t^{EHP} variable is introduced. It expresses the amount of heat produced by the heat pump at the given time. The minimum value of the heat produced is determined by the $H_t^{\min}$ variable and the maximum value is determined by the $H_t^{\max}$ variable. The I_t^h term in the equation determines the operating mode of the heat pump at the given time. In the heat production operating mode, when the value of the I_t^h variable equals to 1, the H_t^{EHP} variable is limited by the $H_t^{\min}$ and $H_t^{\max}$ variables. When the heat pump is not in the heat production operating mode, the value of the I_t^h variable equals to 0 and thus the value of the H_t^{EHP} variable also equals to zero.

Heat pump cooling performance

$$C_t^{\min} I_t^c \leq C_t^{EHP} \leq C_t^{\max} I_t^c. \tag{55}$$

It can be assumed that the heating performance of the heat pump and also the cooling performance of the heat pump are limited. The minimum value of the produced cold is therefore given by the $C_t^{\min}$ variable and the maximum value is given by the $C_t^{\max}$ variable. The amount of cold produced at the given time, C_t^{EHP}, is between these two values. The I_t^c determines whether the heat pump is in the cold production operation mode or not. When the heat pump is in the cold production operating mode, the I_t^c equals to 1, and the C_t^{EHP} variable value is then limited by the values of the $C_t^{\min}$ and $C_t^{\max}$ variables. When the heat pump is not in the cold production operating mode, the value of the I_t^c variable equals to 0 and thus the value of the C_t^{EHP} variable also equals to zero.

5. Initial mathematical model of EnergyHub in GAMS

5.1 Proposal of the optimal EnergyHub model for a specific area

The location in Prague (Czech Republic) was chosen for the design of the optimal EH model (Fig. 7). The buildings in this locality were built between 1920 and 1960.

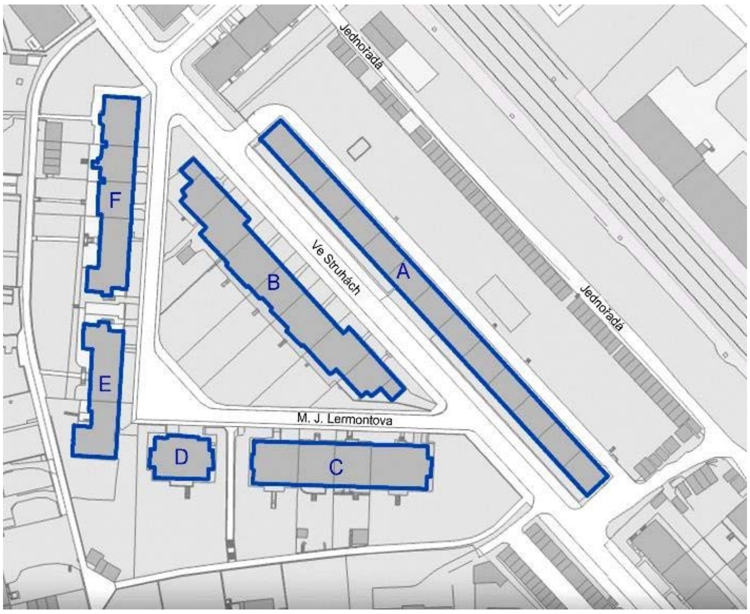


Fig. 7 *Layout and buildings in the specified area of Prague.*

Hourly heat consumption for heating is determined from the parameters of buildings and temperatures and is summarized in the following Tab. I.

Based on EH Fig. 7, then such a structure makes it possible to satisfy the demand for a specific type of energy in various ways. This leads to improved reliability of energy supply and increased degree of freedom in supply – demand issues. Increasing the degree of freedom on the supply side by the possible use of more energy carriers or components within the EH creates room for possible optimization. EH inputs and components can be characterized on the basis of price, emissions, availability and other criteria that allow to optimize the use of energy sources and EH components.

Building	Heat transfer coefficient U_{em} [W/m^{2}K]	Envelope area S [m^2]	Exterior temperature t_e [m^2]	Interior temperature t_i [m^2]	Heat loss of the building Q_c [W]	Hourly heat consumption for heating $Q_{VYT,h}$ [kW]
A	1.4	1578.6	−12	20	70 721.28	70.72
B	1.4	1072.8	−12	20	48 061.44	48.06
C,D	1.4	422.1	−12	20	18 910.08	18.91
E	1.2	1182.6	−12	20	45 411.84	45.41
F	1.2	1182.6	−12	20	45 411.84	45.41
Total					228 516.48	228.51

Tab. I *Hourly heat consumption for heating in the area.*

Building	No. of flats	No. of person per flat	No. of pers.	Amount of hot water per person per day [m^3]	Amount of hot water per day [m^3]
A	80	3	240	0.082 m^3	19.68
B	12		36		2.95
C,D	78		234		19.19
E	112		336		27.55
F	80		240		19.68
Total	362		1 086		89.05

Tab. II *DHW consumption in the given area.*

DHW consumption values in the given area are shown in Tab. II, hourly gas consumption is shown in Tab. III and the hourly electricity consumption of the household is shown in Tab. IV.

5.2 Proposed EnergyHub variants for the given area

The default EH model needs to be modified for application in local conditions. In contrast to the model described in the theoretical part Fig. 6, there is no demand for cooling and thus it is possible to leave out components or operating states that ensure the supply of cooling from the EH model. Furthermore, there is a demand for gas associated with the EH. This fact must be taken into account in the model. In this paper, five variants of EH are proposed for the researched area. Variant 1 (V1) of the EH is connected to the distribution of electricity and gas. At the output, there is a demand for heat, electricity, and gas. Variant 2 is identical to

Building	No. of persons	Hourly gas consumption per person	Hourly gas consumption [kW]
A	240		4.8
B	36		0.72
C,D	234	0.02 kW	4.68
E	336		6.72
F	240		4.8
Total	1086		21.72

Tab. III *Hourly gas consumption.*

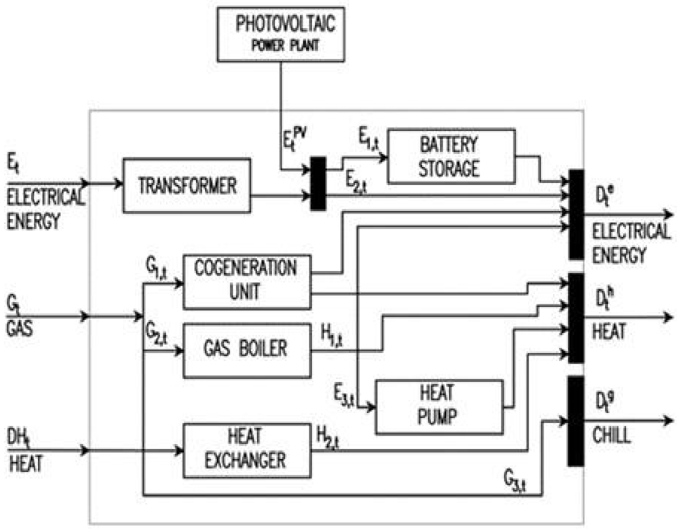


Fig. 8 *Variant 5.*

V1 and is supplemented by input from photovoltaic panels (PP). V3 is exploring the possibility of adding BS to V2. V4 is similar to V3, but the demand for heat is ensured by a heat exchanger instead of a gas boiler. V5 is similar to V3 with the possibility of using heat distribution. After the evaluation, Variant 5 is the best suited EH.

5.2.1 Finding the most optimal variant of EnergyHub using GAMS

From the obtained data from the given area, prepared mathematical models and other described assumptions, the codes of the GAMS program were compiled to find the most optimal variant of EH. I will briefly introduce the variable codes for Variant 5 in GAMS Alg. 2, i.e. for a given time, creating a table of input data and entering input data for consumption, available power of photovoltaic panels and

Annual consumption of the household in the block of flats			2141 kWh
Fig. 7.	TOTAL:	320 households in Ares	
Hour	Coefficient	Sum of coefficients	Household consumption [kW]
1	0.268954		0.15
2	0.2377329		0.13
3	0.2111445		0.12
4	0.2183508		0.12
5	0.2414484		0.13
6	0.2692067		0.15
7	0.3456215		0.19
8	0.4094955		0.23
9	0.4487839		0.25
10	0.4534914		0.25
11	0.4741837		0.26
12	0.4967853		0.28
13	0.4790129	3 865.95	0.27
14	0.4378808		0.24
15	0.467305		0.26
16	0.5120402		0.28
17	0.6165128		0.34
18	0.73712		0.41
19	0.7525153		0.42
20	0.7441204		0.41
21	0.6907377		0.38
22	0.5952402		0.33
23	0.4697944		0.26
24	0.3530899		0.20

Tab. IV *Hourly electricity consumption in a given area.*

hourly price of electricity. Introduction of a variable for the objective evaluation function (8) of the GAMS mathematical model, Alg. 1.

Algorithm 1 Code of the GAMS mathematical model – objective function (9).

```
variable cost ;
*/ zavedeni promenne cost - provozni naklady
```

Introduction of a time variable, creation of a table for input data and input of consumption data, available power of the photovoltaic system, and hourly price for electricity Fig. 9.

I will compile the V5 model, including calling of the solver, see Alg. 3. Variants EH 1 to 5 were investigated and optimized in the environment of the mathematical

```
Set t hours / t1*t24 / ;
*/ vytvoreni promenne t
Table data(t, *)
*/ vytvoreni tabulky data s promennou t
        Dh        De       Dg      PV      lambda_e
t1      422.47    53.92    21.72   0       2.027
t2      422.47    47.66    21.72   0       2.027
t3      422.47    42.33    21.72   0       2.027
t4      422.47    43.77    21.72   0       2.027
t5      422.47    48.41    21.72   0       2.027
t6      422.47    53.97    21.72   0       2.027
t7      422.47    69.29    21.72   0       2.027
t8      422.47    82.1     21.72   0       2.027
t9      422.47    89.97    21.72   14.86   2.027
t10     422.47    90.92    21.72   27.04   2.027
t11     422.47    95.06    21.72   37.7    2.027
t12     422.47    99.6     21.72   24.11   2.653
t13     422.47    96.03    21.72   62.14   2.653
t14     422.47    87.79    21.72   56.01   2.653
t15     422.47    93.68    21.72   43.18   2.027
t16     422.47    102.65   21.72   0       2.027
t17     422.47    123.6    21.72   0       2.027
t18     422.47    147.78   21.72   0       2.027
t19     422.47    150.86   21.72   0       2.653
t20     422.47    149.18   21.72   0       2.653
t21     422.47    138.48   21.72   0       2.653
t22     422.47    119.33   21.72   0       2.653
t23     422.47    94.18    21.72   0       2.653
t24     422.47    70.79    21.72   0       2.027;
*/ vstup hodnot do tabulky data
```

Fig. 9 *GAMS code – input data.*

model GAMS. The results of research in this area show an economic saving of 51.7 %. The results also show the advantages of heat pumps and cogeneration units.

6. Evaluation of results

The results for the individual variants of EH are summarized in Tab. V.

Variant	Value of the objective function
Variant 1	CZK 18,683.95
Variant 2	CZK 18,031.56
Variant 3	CZK 17,708.92
Variant 4	CZK 17,708.92
Variant 5	CZK 17,708.92

Tab. V *Summary of individual energy costs of the individual variants.*

Algorithm 2 Code of variables of V5.

```
positive variables E(t), E1(t), E2(t), E3(t), G(t), G1(t), G2(t),
Ed(t), Ec(t), H1(t), H2(t), H_ehp(t), SOC(t), DH(t), G3(t) ;
*/ introduction of positive variables
binary variables Ih(t), Idch(t), Ich(t) ;
*/ introduction of binary variables
scalar eta_ee / 0.96 / , eta_ge / 0.397 / , eta_gh / 0.527 / , eta_c / 0.9 / , eta_d / 0.9 / ,
COP / 2.47 / , H_ehpMax / 250 / , H_ehpMin / 0.3 / ,Chpmax / 519 / , eta_he / 0.9 / , Fmax / 512 / ,
eta_ghf / 0.915 / , lambda_g / 1.763 / , SOCmax / 232 / , SOC0 / 0 / , lambda_dh / 2.4 / ;
*/ introduction of variables with given values
H_ehp.up(t)=H_ehpMax ;          !! maximum heat pump output
G1.up(t)=Chpmax ;               !! maximum output of the CHP unit
SOC0=0.2*SOCmax ;               !! initial state of battery storage
SOC.up(t)=SOCmax ;              !! maximum capacity of the battery storage
SOC.lo(t)=0 ;                   !! minimum capacity of the battery storage
SOC.fx('t24')=SOC0 ;            !! battery storage cycle
G2.up(t)=Fmax ;                 !! maximum output of the gas boiler
Ec.up(t)=20 ;                   !! maximum charge
Ec.lo(t)=0 ;                    !! minimum charge
Ed.up(t)=130 ;                  !! maximum discharge
Ed.lo(t)=0 ;                    !! minimum discharge
*/ additional specification of variable values
```

Implementation of the EH model can bring real savings in energy costs compared to the current situation.

Report from the solver for variant 5, see Fig. 10.

```
GAMS 30.3.0  rc5da09e Released Mar  6, 2020 WEX-WEI x86 64bit/MS Windows - 05/22/20 20:33:57 Page 5
G e n e r a l   A l g e b r a i c   M o d e l i n g   S y s t e m
Solution Report     SOLVE hub Using MIP From line 123


               S O L V E      S U M M A R Y

     MODEL   hub                  OBJECTIVE  cost
     TYPE    MIP                  DIRECTION  MINIMIZE
     SOLVER  CPLEX                FROM LINE  123

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      1 Optimal
**** OBJECTIVE VALUE            17708.9232

 RESOURCE USAGE, LIMIT          0.016      1000.000
 ITERATION COUNT, LIMIT       108     2000000000

IBM ILOG CPLEX   30.3.0 rc5da09e Released Mar 06, 2020 WEI x86 64bit/MS Window
*** This solver runs with a demo license. No commercial use.
Cplex 12.10.0.0

Space for names approximately 0.01 Mb
Use option 'names no' to turn use of names off
MIP status(101): integer optimal solution
Cplex Time: 0.00sec (det. 1.89 ticks)
Fixing integer variables, and solving final LP...
Fixed MIP status(1): optimal
Cplex Time: 0.00sec (det. 0.56 ticks)
Proven optimal solution.

MIP Solution:        17708.923223    (53 iterations, 0 nodes)
Final Solve:         17708.923223    (55 iterations)
```

Fig. 10 *GAMS – Report of variant 5.*

Algorithm 3 Compiling the V5 model and calling the solver.

```
Equations
eq1 , eq2 , eq3 , eq4 , eq5 , eq6 , eq7 , eq8 ,
eq9 , eq10 , eq11, eq12, eq13, eq14, eq15, eq16, eq17 ;
*/ introduction of the required number of equations with the notation
*/ equations defining mathematical model of the EnergyHub
eq1..     cost =e= sum(t , data(t ,'lambda_e')*E(t)+lambda_g*G(t)+lambda_dh*DH(t)) ;
*/ objective evaluation function - the sum of energy expenditure
eq2(t)..  E2(t)+eta_ge*G1(t)+Ed(t) =e= data(t, 'De')+E3(t) ;
*/ flow of electricity from the EnergyHub
eq3(t)..  eta_ee*E(t) + data(t, 'PV') =e= E1(t) + E2(t) ;
*/ flow of electricity to the EnergyHub
eq4(t)..  E1(t) =e= Ec(t) ;
*/ defining the input of electrical energy into the battery storage
eq5(t)..  SOC(t) =e= SOC0$(ord(t)=1)+SOC(t-1)$(ord(t)>1)+Ec(t)*eta_c-Ed(t)/eta_d ;
*/ change of battery storage charge level
eq6(t)..  Ed(t) =l= 0.2*SOCmax*Idch(t) ;
 */ limiting battery storage discharge
eq7(t)..  Ec(t) =l= 0.2*SOCmax*Ich(t) ;
 */ limiting battery storage charge
eq8(t)..  Idch(t)+Ich(t) =l= 1 ;
*/ operating mode (charging/discharging)
eq9(t)..  eta_ghf*G2(t) =e= H1(t);
*/ operation of the gas boiler
eq10(t)..  G(t) =e= G1(t)+G2(t)+G3(t) ;
*/ Flow of gas to Energy Hub
eq11(t)..  G3(t) =e= data(t , 'Dg') ;
*/ Flow of gas from Energy Hub
eq12(t)..  eta_gh*G1(t)+H1(t)+H2(t)+H_ehp(t) =e= data(t, 'Dh') ;
*/ Flow of heat from Energy Hub
eq13(t)..  eta_he*DH(t) =e= H2(t) ;
*/ operation of the heat exchanger
eq14(t)..  H_ehp(t) =e= E3(t)*COP ;
*/ heat pump operatic
eq15(t)..  H_ehp(t) =l= H_ehpMax*Ih(t) ;
*/ limitation of the maximum heat output of the heat pump
eq16(t).. H_ehp(t) =g= H_ehpMax*Ih(t)*H_ehpMin;
*/ limitation of the minimum heat output of the heat pump
eq17(t).. Ih(t) =l= 1 ;
*/ heat pump operating mode (cooling / heating)
Model hub / all / ;
*/ creating a mathematical model from the entered data
Solve hub us mip min cost ;
*/ command for solving the model with MIP solver with minimization of the variable "cost"
```

The value of the objective function for variant 5 is: CZK 17,708.92. The daily energy costs of the current state amount to CZK 34,233.57. The most economical variants of the EH model show that the daily energy costs could fall to CZK 17,708.92. So the possible saving is CZK 16,524.65.

The results from the examined area show an economic saving of 51.7 %. The results also show the main advantages of heat pumps and cogeneration units. Their high efficiencies and available performance offer great potential for their development. EH was investigated from the point of view of a mathematical model and its optimal functioning. This partial goal was achieved with the help of GAMS software, which represents certain originality in this symbol, which enables the compilation, analysis and solution of complex mathematical models.

The Chart in Fig. 10 shows the course of electricity consumption from the distribution network $E(t)$, the power supplied by photovoltaic panels $PV(t)$, the production of electricity by the cogeneration unit CHP (Combined Heat and Power) and the electricity entering the battery storage $E1(t)$. The implementation is evident in Fig. 8.

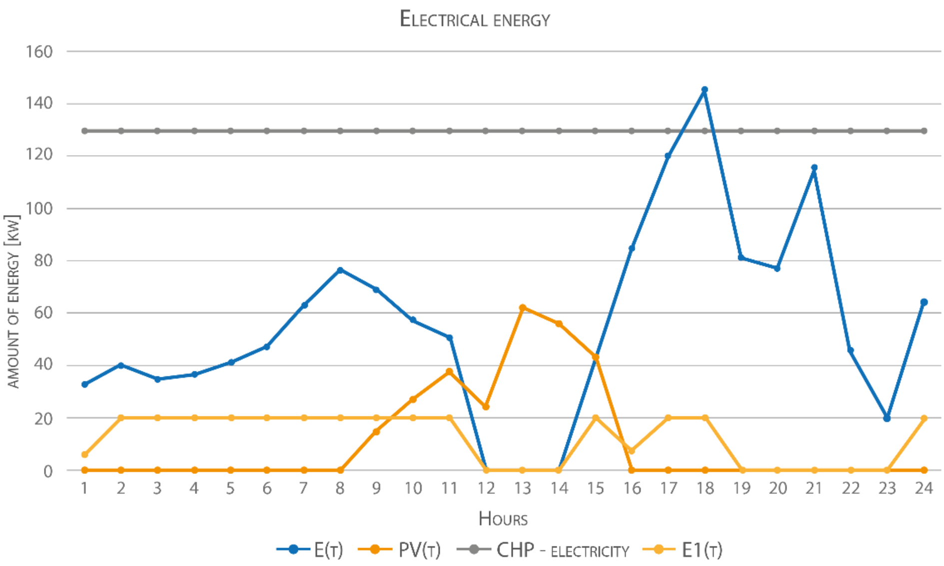


Fig. 11 *Electricity – variant 5.*

6.1 Photovoltaic energy, simulation, optimization

Suggested number of photovoltaic panels based on the building layout in Fig. 7. The essence of the energy concept of the area, Fig. 7, is the interconnection of photovoltaic systems from the whole area (FV1, FV2, FV3, FV4, FV5, FV6) into one large system, including the cogeneration energy system with the power plant in the TS-DS 22 substation/0.4 kV; EH see Fig. 8, in the middle of the intelligent area. Such a system responds more flexibly to the current need for electricity in the sub-buildings of the cluster of buildings and at the same time provides energy according to the plan for optimizing the allocation of resources directly in the area [3]. This eliminates the problem that would arise with separate systems. In Tab. VI shows the number of photovoltaic panels, their outputs and annual electricity production.

Despite the relatively large roof areas in the area, the space for installing photovoltaic panels is very small: due to the presence of physical obstacles – ventilation shafts, chimneys, lightning conductors, satellite dishes; or placement of buildings in the main directions – the main facades of buildings face north or northeast.

HOMER (Hybrid Optimization Model for Electrical Renewable) is software that is used to simulate and optimize possible combinations of used renewable energy sources and their use. In the case of this project Fig. 12, the simulation results are shown so that the brown color of the column shows the energy, the green color of

Building	Roof area [m^2]	Available area [m^2]	Direction	Slope [°]	Number of panels	Output [kWp]	Annual production [kWh]
A/FV1	888	800	SV	30	400	100	95 000
B/FV2	1 215	730	SV	15	365	91.25	86 688
C/FV3	850	350	S	15	170	42.5	40 375
D/FV4	265	100	S	15	53	13.25	12 588
E/FV5	617	180	W	30	90	22.5	21 375
F/FV6	921	285	W	30	142	35.5	33 725
				Σ	1 220	305	289 751

Tab. VI *Suggested number of PV panels (FV1 to FV6: photovoltaics).*

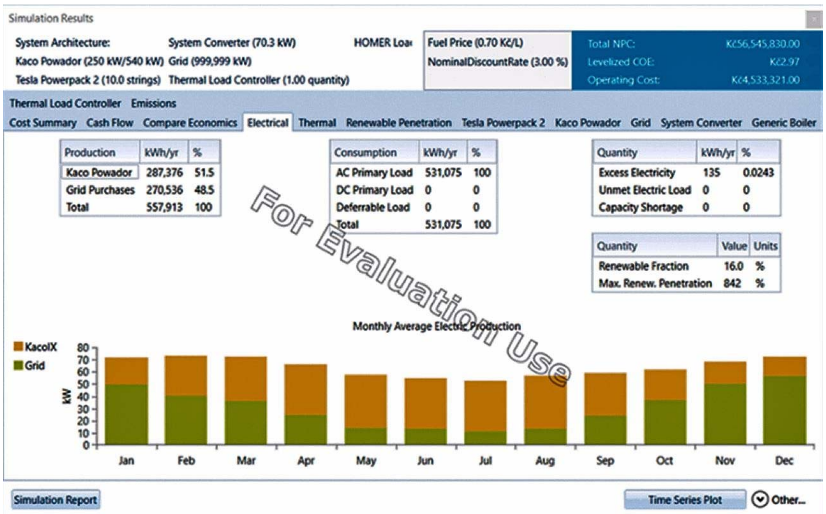


Fig. 12 *Simulation results. Yellow = photovoltaic energy, green = grid energy.*

the column shows the energy from the network. Therefore, the most advantageous in terms of the use of renewable energy is the use of PVE, cogeneration batteries. In this case, 51.5 % of electricity is covered annually. Fig. 12 shows the importance of the application of a photovoltaic power plant (PPP), ie the consequence of the electrical coverage of the territory in comparison with the electricity from the distribution network.

7. Evaluation of results

The results show that the use of a photovoltaic system and battery storage has reduced the daily energy costs of EH. The introduction of the EH model can bring real savings in energy costs compared to the current situation.

Thermal and electrical loads were entered for optimization. Furthermore, types of renewable energy sources (PS – photovoltaic system, cogeneration), battery storage, central heat source DH_t and central source of electricity E_t.

The results for the individual variants of EnergyHubs are summarized in Tab. V. The results show that the use of a photovoltaic system and battery storage has reduced the daily energy costs of EnergyHub. The introduction of the EnergyHub model can bring real savings in energy costs compared to the current situation.

The daily energy costs of the current state amount to CZK 34,233.57; the most economical variants of the EnergyHub model show that the daily energy costs could fall to CZK 17,709.92. So the possible saving is *CZK 16,524.65*

Variants 3, 4, 5 (only described verbally, see Tab. V) have the same daily energy costs due to the fact that the demand for thermal energy is covered mainly by the production of heat from the heat pump. The heating factor of the heat pump, even at such low temperatures, provides sufficient output and thus an economic advantage for the use of the heat pump both in EH and for normal heating and DHW preparation in family houses.

The heat pump works at maximum output throughout the day and the remaining heat energy is produced by a cogeneration unit. It is clear from this that the production of heat by burning gas in a gas boiler or the use of district heat to meet the demand for thermal energy is not economically advantageous compared to the use of heat pumps to cover this demand.

As the heat pump is a very complex device that can be prone to failure, it would be very practical to leave a backup source of heat energy in the EH, either in the form of a gas boiler or a heat exchanger connected to the heat distribution network.

When examining the results, it is also necessary to take into account the fact that these variants are calculated for the most unfavourable day of the year. If other days (such as summer days) were examined, the results would be different. Other possible costs associated with the operation of the EH are also not included; only the energy costs are included in the calculations. The most objective results would be achieved in the case of modelling the operation of EH variants throughout the year, by including and comparing other costs associated with the operation of EH. However, this is no longer included in this work due to its scope.

Maximizing the use of available RES is an important starting point for the proposed solution. The installation of photovoltaic panels in summer covers up to 80 % of the energy needed for the household.

The conclusion concerning the reduction of emissions, reached in the context of this research and implementation is very important: Based on Tab. VI, the electricity consumption for one household is: 2141 kWh/year. Then the total electricity consumption for 320 households in the specified area according to Fig. 7 is 685,120 kWh/year.

Since the coefficient of contemporaneity at the level of 0.6 must be used according to the Czech standard, then the total electricity consumption in the area is 411,072 kWh/year. The total electricity produced by the photovoltaic system (PVS) installed on the roofs of individual housing clusters in the given area according to Tab. VI is 289,751 kWh/year.

Due to an estimate of the amount of CO^2 (in kg) emitted into the atmosphere due to electricity production based on a national energy mix, the emission coefficient was assumed to be 283.6 g CO^2/kWh)[1]. This leads to the following conclusion:

1. With the electricity consumption in a given area of 411,072 kWh / year, *CO^2 production is equal to 116.58 t of CO^2/year.*

2. In case of application of RES installation in the given area, the total consumption of electricity supplied from the distribution network will be at the level of 121,321 kWh/year, which means the production of CO^2 would be 34.4 t of CO^2/year.

3. Due to the installation of PVS in the area and its optimal design using PV*SOL software, *reduction in CO^2 emissions by 29.5 %* will be achieved.

4. The introduction of the EnergyHub model can bring real savings in energy costs compared to the current situation. The daily energy costs in the given area of the current state amount to *CZK 34,233.57*; the most economical variants of the EH model in Tab. V show that daily energy costs could fall to CZK 17,708.92. Thus, the possible savings using the most economical variant of the EH model in Tab. V are 16,524.65, i.e. *51.7 %*. This corresponds to a reduction in energy consumption using GAMS software optimization.

5. Based on the above calculation and the application of energy sustainability, with a significant reduction in CO^2 emissions, a total emission reduction of 81.2 % will be achieved.

The EH was investigated from the point of view of a mathematical model and its optimal functioning. This sub-goal was achieved using GAMS software, which makes it possible to compile, analyse, and solve complex mathematical models. The results also show the main advantages of heat pumps and cogeneration units. Their high efficiencies and available performance offer great potential for their development. The same applies to the installation of PVS in the area.

Recommendations and opinion of the researcher In this research, the EH model is presented with load uncertainties (electricity, heating and cooling) and electricity prices to optimize energy costs. However, renewables with stochastic characteristics and investment costs for equipment are not taken into account. I recommend that the above aspects be further explored in solving the problems of optimal planning and operation for EH.

[1]https://ec.europa.eu/environment/emas/pdf/other/ES2016_Consolidated{%}20version_final_en.pdf;
https://www.svetmobilne.cz/emise-co2-u-elektromobilu-tesla-horsi-nez-bmw/4645-2;
https://www.isprambiente.gov.it/files/pubblicazioni/rapporti/R_212_15.pdf (ISPRA 2015)

8. Conclusion

Our experiment – research addresses the following key issues:

1. The most suitable model of EH variant 5 with different energy load capacity is proposed. Commercial use of large system devices for energy storage using batteries (on-grid) (Battery Energy Storage Systems, hereinafter "BESS") is limited, due to insufficient setting of legal conditions for the operation of such devices in the Czech Republic. The proposed model takes BESS into account to increase operational flexibility and efficiency. This results in improving economic and technical aspects of EH.

2. EH's operational plans shall be optimized while respecting the minimum cost of energy purchased from contractors, taking into account the uncertainties of energy demand and electricity prices.

3. The mathematical model EH represents the uncertainties of the three types of loads (electricity, heating, cooling) and the price of electricity. The results show that the energy flow is optimized in all cases, at the same time the technical requirements for the equipment are ensured. Taking into account uncertainties, demand and total energy purchased from contractors will be reduced. Therefore, the calculation error decreases and the results are more suitable for problems in real conditions.

In my research, the EH model is presented with load uncertainties (electricity, heating and cooling) and electricity prices to optimize energy costs. Renewable energy sources in our case photovoltaic energy and investment costs for equipment are not taken into account. Then the above aspects should therefore be further explored in the problems of optimal planning and operation for EH.

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