OPTIMIZATION APPROACH FOR HANDLING IRROPS FROM THE PERSPECTIVE OF PASSENGER IMPACT

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Abstract: The paper focuses on optimization approaches in the operational management of airlines, emphasizing the impacts of irregular operational situations (IRROP) on passengers. The introductory part describes the complexity of the business models of airlines and highlights the importance of operational management in minimizing the negative effects of IRROP. The historical analysis shows the evolution of approaches from reactive management concerning only costs aspect of IRROP to predictive management considering the impact on passengers as main factor, incorporating modern methods and collaborative approaches. Furthermore, the necessity of objective decision-support tools that minimize the impact on passengers and increase passenger satisfaction is discussed. The main goal of the paper is to identify key factors affecting the daily utilization of the aircraft fleet and to propose fuzzy linear programming as an effective method for optimizing operations.

Key words: *irregularity operations, IRROP, predictive operation management, airline fleet optimization, mathematical programming, fuzzy numbers, linear programming with uncertainty*

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1. Introduction

The business model of airlines operating scheduled (regular) and unscheduled (charter) passenger or cargo transport is based on the transportation of passengers or shipments by aircraft on selected routes throughout the period of the flight schedule's validity.

In general, the business model and the associated real transport process of an airline can be defined as a complex and intricate system of activities that ensure the transportation of passengers or cargo and include the following activities:

– sales of products to customers,

– check-in of passengers or air cargo at airports,

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– flight planning,

– crew scheduling,

– technical service (maintenance) planning for aircraft,

– execution of passenger and luggage transport or air cargo transportation.

The structure of an airline's business model is a highly complex process, and in addition to the requirement to adhere to all planned procedures, it also necessitates preparedness for irregularities in operations (IRROP) that may arise. The occurrence of IRROP in operations is primarily caused by an extreme increase in traffic density, as dense traffic is much more prone to the occurrence of IRROPs.

2. Operational Management as a Tool for Managing Operational IRROP

Dealing with the consequences of IRROP falls under the remit of the so-called operational management. Operational management is currently gaining considerable importance and the impact of its work on the overall economics of air carrier operations is considerable. The current state of operational management can be characterized as a transition from dealing with the "What happened?" system to a "What happens if. . . ?" system. Operational management currently produces coordinated solutions that, while maintaining the economics of operations, primarily minimize the impact on passengers of the consequences of an IRROP.

Flight operational control is the responsibility of the Operational Control Center (OCC). The OCC is tasked with managing and coordinating the implementation of the daily schedule as planned, with maximum emphasis on optimizing adherence to the daily schedule, while maintaining the required quality of service. The OCC's domain is primarily dispatching interventions in the organization of operations when an IRROP occurs. The main objective of dispatching interventions in the event of an IRROP is to optimize the utilization of the aircraft fleet while managing the number of passengers transported, whose carriage will not be denied by the IRROP.

In the 1970s and 1980s, when the first OCCs were formed, this was a way of working based on a system of 'firefighting' or working out what to do in the event of an IRROP being formed. The effort was to solve the problem in a way that focused on dealing with just that one IRROP. At the same time, the culprit was always sought, and naming or punishing him was considered an important preventive element of the procedure. The result of this approach was that individual airline departments had their own metrics for dealing with the IRROPs that arose, and their performance varied widely. The cost of the solution was not taken into account, nor was the impact on customer satisfaction, i.e., passenger satisfaction. This operational management system is called reactive.

From the 1990s until the end of the century, the management system evolved considerably towards more systemic solutions. Scenarios were planned in advance to address the possible types of IRROPs, with a view to a balanced solution from both an air traffic and maintenance perspective, and coordinated solutions and decisions led to close cooperation between the various airline departments involved. The aim of all was then to provide the fastest and most coordinated solution possible, with the performance of the different sections balanced. However, the solutions were often sub-optimal, especially in terms of the direct impact on passengers. This system of operational management is called proactive.

From the beginning of the 21st century until today, the operational management system has moved in this direction. The element of preventive coordinated management of IRROPs has remained, but with maximum regard to the total costs associated with the solution of the IRROP and with the least possible decrease in passenger satisfaction, where the cost impact includes not only the increase in the cost of the specific solution (e.g., rescheduling, accommodation, compensation) but also the decrease in the number of potential customers based on their bad experience of the solution of the IRROP and the overall media image of the air carrier, which is now considered to be a very important view of the success of the solution. In these respects, then, the analytical solutions and planning are coordinated, and the management system is considered open to new innovations and continuous development. We call this operational management system predictive.

Air carriers pursuing modern management methods are currently moving to this type of predictive operational traffic management, which seeks the optimal solution to an IRROP arising in terms of minimizing disruption to the flight schedule, reducing the cost of resolving the IRROP whilst minimizing the impact on passengers.

The main tasks of the OCC can therefore be summarized as actively monitoring and managing the accuracy of the daily traffic plan and when an IRROP occurs subsequently:

- pro-actively manage the impact of unplanned changes on the original plan,
- to provide the required quality of service to the maximum number of passengers,
- achieve the original plan with the least impact on the total number of passengers carried.

The OCC can be designed either as a separate department or as an integrated system (so-called IOCC – Integrated Operation Control Center). The stand-alone OCC, typical of small carriers, where all interactions between the various departments involved are usually carried out by telephone, is being replaced by large "open space" workplaces where the staff involved are co-located with the possibility of direct interaction.

Currently, the dominant approach to addressing an IRROP is a human-subjective perspective on the situation, although this is often based on the use of partial software tools. An objective tool or system to support decision-making could address the shortcomings of the existing system, which is largely based on experience but still heavily reliant on subjective perceptions of the consequences of the IRROP. An objective decision-support tool or system could significantly minimize both the economic and operational impact when resolving IRROPs occurring during the daily operations of airline carriers.

Since operational management should be carried out efficiently and in the shortest possible time, preparedness for the occurrence of IRROPs in operations is crucial. Ideally, this involves developing solution scenarios for possible types of IRROPs, which would significantly accelerate and improve the work of the operational control dispatcher.

3. Analysis of State-of-the Art

The publication [1] provides a global view on the issue of dealing with emerging IRROPs. The publication considers collaborative management as the main tool for dealing with emerging IRROPs. Collaborative management is intended to be based on the application of information sharing and distributed decision making principles to air traffic flow management (ATFM) expanding the availability of databases and their use by operations management personnel (movement control and slot coordinators), establishing common situational awareness, introducing shared tools and procedures in real time, and increasing the importance of planning for robustness and reliability of the air traffic order to the level of cost minimization in the air traffic environment.

The next paper in this group is by [2]. The main focus of this publication is to summarise the state of practice of the Air Operations Control Center (AOCC) during the handling of the aftermath of an IRROP. An overview of the structure of a typical AOCC is provided as a basis for discussion. The main causes of IRROPs are considered based on operational data from the US domestic market. An overview of current information and decision support systems used in AOCCs by US air carriers is presented.

Options for addressing sub-problems arising from the occurrence of IRROPs are also included. The paper [3] addresses the issue of minimizing the total tactical costs incurred by airport departure delays. The input data are sets of input parameters such as the set of affected aircraft, the set of aircraft turnaround activities, passenger disembarkation and embarkation, refuelling, aircraft cleaning and related activities, baggage and parcel unloading and loading, etc. In addition, the input parameters include, for example, the total cost of turnaround, the cost of cancelling a passenger's transport (cost of rerouting the flight, accommodation costs and compensation arising from the legislation currently in force in the event of longer delays), the number of coaches for passenger transport, the number of staff involved in the aircraft turnaround, the turnaround time of the aircraft, etc. The optimisation criterion is the total delay for the turnaround of a group of aircraft at a transit hub. The computational experiments were performed at Frankfurt Airport for 20 departures in the morning peak hour in the time interval 7:30–11:00.

The work of [7] deals with the development of an optimization model, but again only for a subsystem of operational control. Specifically, it deals with the problem of managing the movement of airline crews during the emergence of IRROP, since the authors of the paper consider crew movement as the bottleneck of the whole process of restoring system functionality. The bottleneck, according to the authors, is caused by the complexity of crew schedules resulting from restrictive active duty rules and the size and scope of the hub-and-spoke line networks of large air carriers. Inputs are aircraft numbers, crew numbers, flight numbers, and restrictive crew workload standards. The optimization approach proposed in their paper allows decisions to be made about the number of crew exchanges when an IRROP occurs and the total number of crews required to cover the range of traffic under study. Extensive testing was performed with the proposed optimization approach in dealing with different types of IRROPs based on real data, and sufficient effectiveness of the proposed approach was confirmed.

The work of [6], describes a situation where one or more aircraft from the fleet is retired due to technical reasons and the air carrier has to operate in the existing network with a reduced number of aircraft. This paper presents the results of an effort to define a network graph for this situation so as to minimize the total passenger delay on the air carrier's route network. The resulting network in which the nodes represent flights on a given airline network and the edge evaluation is the time loss on each flight. The solution of the problem is based on the branch and bound method.

In the work of [4] the creation of a new daily aircraft usage plan in case of emerging IRROPs. The model also includes the issue of crew changes, aircraft maintenance. They use a rather complicated network model to solve it. Their network consists of a set of nodes where each pair is connected by a directional arc representing each flight segment. Additional directed arcs are then used to represent other aircraft movements for maintenance and ground operations, crew movements to move between the aircraft and the resting place, as well as connections for passengers.

Also very inspiring is the work of [5], in which the author discusses the use of fuzzy logic and fuzzy numbers in solving many areas of optimization in different types of transportation. The author outlines the potential of the fuzzy approach. Fuzzy logic has the potential to effectively model situations in which people make decisions in extremely complex environments where it is difficult to create an accurate mathematical model. These situations are commonly encountered, for example, in the transportation field when analyzing the work of dispatchers or modeling selection problems. Current experience suggests that various algorithms for approximate reasoning exist for solving such complex problems. This paper focuses on the classification and analysis of the results obtained using fuzzy logic in modelling complex traffic processes. Fuzzy logic appears to be a very promising mathematical method for modeling traffic processes that are affected by subjective beliefs, ambiguity, uncertainty and imprecision. In this paper, the basic principles of fuzzy logic and a detailed analysis of various fuzzy logic systems that have been developed to solve specific problems in the field of traffic engineering are presented. Emphasis is placed on the importance of these systems as universal approximation approaches for solving transportation related problems. The possibilities of further applications of fuzzy logic in this area are also presented.

By analysing the publications published so far, the works that have been identified in the past that dealt with the problem of solving the emerging IRROPs in air passenger transportation, it was found that the works that provide an overall view of the problem propose solutions based on CDM systems or most of the works deal with the problem only from the operational perspective of the carrier. None of the papers addressed the IRROP issue from the perspective of its direct impact on the number of passengers carried.

4. Design of the Optimization Approach

The optimisation problem arises in a situation where an IRROP of such magnitude occurs that there is an undesirable impact on the flight plan in the near future with a possible direct impact on the number of flights served and therefore a direct impact on the number of passengers carried.

The set of flights I to be served in the event of an IRROP is given. For each flight $i \in I$, the earliest planned start time of pre-flight activities t_i, the duration of the flight including the times required to perform all pre-flight and flight activities T_i, the maximum delay time of the start time of pre-flight activities a_i and the number of passengers who have purchased a ticket c_i for flight $i \in I$ are defined. For each pair of flights $i \in I$ and $j \in I$, where $i \neq j$, an unproductive so called positioning or deadheading time τ_{ij} is defined after the operation of flight $i \in I$ to the operation of flight $j \in I$ (including cases where the destination at which the flight ends is also the departure airport of the following flight). Information is also available on the total number of N aircraft available to the carrier and the number of K aircraft affected by IRROP.

Suppose further the carrier has a homogeneous fleet (all operational aircraft in the fleet are interchangeable for all flights), the IRROP is detected at the beginning of the operating day, and the IRROP will take the entire operational day to clear.

The challenge is to optimise the daily fleet utilisation, i.e., to schedule the deployment of aircraft so that the total number of passengers allowed to fly on a given operating day is maximised. Scheduling aircraft deployment under homogeneous fleet conditions means deciding on the sequences of flight operations by each aircraft.

To model elementary decisions in the optimization problem, we introduce a set of binary variables x_{ij}, for $i \in I \cup \{0\}, j \in I \cup \{0\}$, with $i \neq j$, whose domain consists of values 0 and 1. These variables will model the decision that the same aircraft, after serving flight $i \in I$, will be assigned to serve flight $j \in I$, where $j \neq i$. A value of $x_{ij} = 1$ at the end of the optimization process represents the decision that flight $j \in I$ is scheduled in the aircraft's daily operation plan immediately after flight $i \in I$, where $i \neq j$. Conversely, a value of $x_{ij} = 0$ indicates that flight $j \in I$ will not be scheduled immediately after flight $i \in I$ in the aircraft's daily plan. Among the variables x_{ij}, where $i \in I \cup \{0\}$ and $j \in I \cup \{0\}$, the variables x_{0j} for $j \in I$, and x_{i0} for $i \in I$, have a specific role. If the binary variable $x_{0j} = 1$ after the optimization process, then flight $j \in I$ is the first flight operated by the specific aircraft on the given day. If $x_{0j} = 0$, then flight $j \in I$ is preceded by another flight operated by the same aircraft. Analogously, the variables x_{i0} carry the meaning that if $x_{i0} = 1$, then flight $i \in I$ is the last flight operated by the aircraft on that day. If $x_{i0} = 0$, then the aircraft will operate at least one more flight after flight $i \in I$. Since it is permissible for the start time of pre-flight procedures a_i, where $i \in I$, to be delayed, we also introduce a set of non-negative variables z_i, where $i \in I$. Each variable z_i, with domain R_0^+, represents the decision concerning the possible delay (measured from scheduled time t_i) in starting the pre-flight procedures required for serving flight $i \in I$.

The mathematical model of the problem is given as

$$\max \sum_{j \in I} c_j \cdot \sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij}, \tag{1}$$

subject to:

$$\sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij} \leq 1, \qquad \text{for } j \in I, \tag{2}$$

$$\sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ji} = \sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij}, \qquad \text{for } j \in I, \tag{3}$$

$$\sum_{j \in I} x_{0j} \leq N - K, \tag{4}$$

$$t_i + T_i + z_i + \tau_{ij} \leq t_j + z_j + W \cdot (1 - x_{ij}), \quad \text{for } i \in I \cup \{0\}, j \in I \cup \{0\}, j \neq i, \tag{5}$$

$$z_i \leq a_i, \qquad \text{for } i \in I, \tag{6}$$

$$z_j \leq M \cdot \sum_{i \in I \cup \{0\}} x_{ij}, \qquad \text{for } j \in I, \tag{7}$$

$$x_{ij} \in \{0; 1\}, \qquad \text{for } i \in I \cup \{0\}, j \in I \cup \{0\} \text{ and } j \neq i, \tag{8}$$

$$z_i \in R_0^+, \qquad \text{for } i \in I. \tag{9}$$

Function (1) represents the optimization criterion—the total number of passengers whose flights will be served by an aircraft. The group of constraints (2) ensures that at most one aircraft is assigned to serve each flight. The group of constraints (3) ensures that if flight $j \in I$ is served, it will be followed either by the service of another flight $i \in I$ or the flight will be the last in the defined planning period for a specific aircraft. Constraint (4) ensures that no more aircraft are assigned to flight service than the carrier has available. The group of constraints (5) ensures that if the service of flight $j \in I \cup \{0\}$ after flight $i \in I \cup \{0\}$ when $j \neq i$ is not temporally feasible, it will not be allowed either. The group of constraints (6) ensures that the delay in the start time of pre-flight activities at the departure airport for flight $i \in I$ does not exceed the point in time that would cause a unacceptable flight delay. The set of constraints (7) ensures that when a flight is not operated, the variable modeling the shift in the start of preflight preparation will be equal to 0. Groups of constraints (8) and (9) define the domains of definition of the variables used in the proposed model. The prohibitive constant W from group of constraints (5) has the task of covering the amount on the left side when it applies $t_i + z_i + T_i + \tau_{ij} > t_j + z_j$ for any pair of flights i and j, when $j \neq i$. When this applies to any pair, the worst case scenario occurs for the maximum value $t_i + z_i + T_i + \tau_{ij}$ and the minimum value $t_j + z_j$. The quantities t_i, T_i, τ_{ij} are constants, the quantities z_i are variables ranging in the interval $\langle 0; a_i + p_i \rangle$. We therefore determine the constant W as

$$W > \max_{i \in I \cup \{0\}} \{t_i + z_i + T_i + \tau_{ij}\} - \min_{j \in I, j \neq i} \{t_j + z_j\}.$$

However, some of the input values included in the mathematical model may be subject to uncertainty in real-world operations. Let us assume in our further considerations that the coefficients a_i, for $i \in I$, are subject to uncertainty. A possible cause of this uncertainty may be, for example, a longer ground handling (turnaround) time than originally planned. Therefore, the maximum time allowing for a shift in the start of pre-flight preparation for flight $i \in I$, denoted a_i, will be approximated by a fuzzy set of values approximately less than the crisp value a_i, described by the membership function $\mu_{\leq a_i}(y_i)$ in the form:

$$\begin{aligned} \mu_{\leq a_i}(y_i) &= 1, && \text{for } y_i \leq a_i, \\ \mu_{\leq a_i}(y_i) &= \tfrac{a_i+p_i-y_i}{p_i}, && \text{for } y_i \in (a_i; a_i+p_i), \\ \mu_{\leq a_i}(y_i) &= 0, && \text{for } y_i \geq a_i+p_i, \end{aligned} \tag{10}$$

where the y_i is real value represents the possible extension time and the value p_i represents the maximum acceptable extension time a_i. The defined membership function (10) corresponds to the graphical representation in Fig. 1.

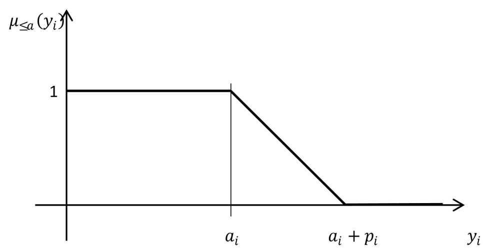

Fig. 1 *Graphical representation of the fuzzy number y_i membership function.*

When the input variables are uncertain, the value of the objective function (1) is also uncertain. Let us denote a particular value of the objective function by f. Since the objective function (1) is being maximized, we approximate it using a fuzzy set of sufficiently large values (large), defined by a membership function $\mu_{\text{large}}(f)$.

$$\begin{aligned} \mu_{\text{large}}(f) &= 0, && \text{for } f \leq F^{\min}, \\ \mu_{\text{large}}(f) &= \tfrac{f-F^{\min}}{F^{\max}-F^{\min}}, && \text{for } f \in \left(F^{\min}; F^{\max}\right), \\ \mu_{\text{large}}(f) &= 1, && \text{for } f \geq F^{\max}, \end{aligned} \tag{11}$$

where $F^{\max}$ is the value of the objective function obtained under the most favorable conditions, and $F^{\min}$ is the value of the objective function obtained under the least favorable conditions. The least favorable condition is considered to be the operating scenario shown in Fig. 1, where $x = a_i$, and the most favorable condition is represented by the scenario in Fig. 1, where $x = a_i + p_i$. The fuzzy set of sufficiently large values, defined by the membership function (10), is graphically represented in Fig. 2.

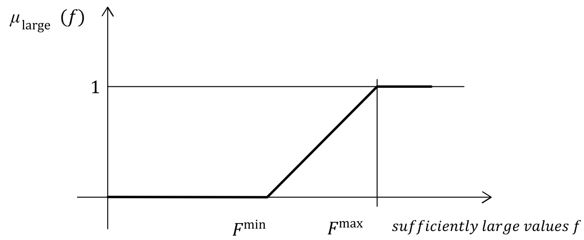

Fig. 2 *The membership function of sufficiently large values of the objective function f.*

If the calculated value of the objective function f after completing the optimization computation satisfies $f \geq F^{\max}$, then this value is considered sufficiently large at a level of satisfaction of 1 (we are fully satisfied with the result). If $f \leq F^{\min}$, the calculated value is considered sufficiently large at a level of satisfaction of 0 (we are not satisfied with the result). If $f \in (F^{\min}; F^{\max})$, then the value is considered sufficiently large at a corresponding level of satisfaction $h \in (0; 1)$, and we are partially satisfied—the degree of satisfaction increases with a higher level of satisfaction. The calculation of $F^{\min}$ will be considered as Phase 1 of the optimization calculation. The calculation of the $F^{\max}$ value will be considered as Phase 2 of the optimization calculation. In order to further solve the optimization problem, we introduce a variable h modeling the level of satisfaction of the found solution and in Phase 3 of the optimization calculation we solve the model in the form:

$$\max h, \tag{12}$$

subject to the constrains (2)–(5), (7)–(9) and also these constrains:

$$z_i \leq h \cdot a_i + (1-h) \cdot (a_i + p_i), \qquad \text{for } i \in I, \tag{13}$$

$$\sum_{j \in I} c_j \cdot \sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij} \geq h \cdot F^{\max} + (1-h) \cdot F^{\min}, \tag{14}$$

$$h \leq 1, \tag{15}$$

$$h \in R_0^+. \tag{16}$$

Function (12) represents the optimization criterion—the level of satisfaction at which the transportation of a sufficiently large number of passengers whose flights were dispatched (taking into account acceptable delays) is achieved. The group of constraints (13) ensures the acceptability of shifting the planned start time of pre-flight activities for the service of flight $i \in I$. The interpretation of the right-hand sides of these constraints depending on the level of satisfaction is as follows: when $h = 1$, any shift in the planned pre-flight time is due only to the usage of the planned time buffer according to the current flight schedule. For $z_i \in (a_i; a_i + p_i)$, the level of satisfaction of the obtained solution decreases. If $z_i = a_i + p_i$, the solution's level of satisfaction becomes $h = 0$. Constraint (14) establishes the relationship between the achieved level of satisfaction and the total number of passengers whose flight was dispatched (taking into account acceptable delays). Constraint (15) ensures the calculated level of satisfaction does not exceed the maximum possible value of the membership function. The group of constraints (16) defines the domain of the variable modeling the level of satisfaction.

However, the type of inequality in constraint (14) combined with the objective function (12) does not guarantee finding a solution containing the maximum number of passengers carried. Therefore, it is desirable to continue the solution, fix the calculated value of h and then, with the fixed value of h, maximize the total number of passengers carried, i.e., solve the model containing the function (1) again under constraints (2)–(5) , (7)–(9) , (13)–(14) substituting the value calculated in Phase 3 for h in constraint (14). Let us denote this procedure as Phase 4 of the optimization calculation.

For this reason, it is desirable to continue the optimization calculation of Phase 5 and to modify and solve the mathematical model (1), (2)–(5), (7)–(9) and (13)–(14) so that the above-mentioned unproductive positioning flights in the design do not occur. Excess unproductive positioning flights will be quantified by the total time of unproductive positioning flights. The model of the optimization problem solved in Phase 5 of the optimization calculation has the form:

$$\min \sum_{i \in I \cup \{0\}} \sum_{\substack{j \in I \cup \{0\} \\ j \neq i}} \tau_{ij} \cdot x_{ij}, \tag{17}$$

subject to constraints (2)–(5), (7)–(9) and (13)–(14), where under constraints (13) and (14) the value of h is substituted with the value computed in solving model (2)–(5), (7)–(9) and (12)–(16) in Phase 3 and a condition is added to ensure that the total number of passengers carried does not fall below the value calculated in Phase 4. Assume that the total number of passengers carried in Phase 4 is P. The constraint ensuring that the total number of passengers carried does not fall below the value calculated in Phase 4 is of the form:

$$\sum_{j \in I} c_j \cdot \sum_{\substack{i \in I \cup \{0\} \\ i \neq j}} x_{ij} \geq P. \tag{18}$$

Function (17) represents the optimization criterion—the total duration of non-productive overflights required to transport at least P passengers, where P corresponds to the right-hand side of constraint (14) at the level of satisfaction h calculated in model (1)–(5), (7)–(9), (13) and (14) in the Phase 4.

However, in the model solved in Phase 5 of the optimization calculation, in which the value of the total time of unproductive positioning flights is minimized, redundant time delays of the start of preflight preparation times can be ordered through the values of the variables z_i. To this end, it is desirable to continue the optimization calculation with Phase 6, in which the model will be solved in the form:

$$\min \sum_{i \in I} z_i, \tag{19}$$

under constraints (2)–(5), (8)–(9) and (13), (14) and (16), where the value of h in constraint (14) will be replaced by the value calculated when solving the model

(2)–(5), (7)–(9) and (12)–(16) in Phase 3 of the optimization calculation. To these constraints will be added constraint (19) of the form:

$$\sum_{i\in I\cup\{0\}} \sum_{\substack{j\in I\cup\{0\}\\ j\neq i}} \tau_{ij} \cdot x_{ij} \leq R, \tag{20}$$

where the symbol R represents the value of optimization criterion (17) computed in Phase 5.

Finally, the optimization approach will be recapitulated. The optimization approach consists of six successive phases and it is recapitulated in Tab. I.

Solution phase and brief description	Obj. funct.	Constraints
1. A pessimistic evaluation of the optimization criterion, based on a model solution using input data that reflect the most adverse operational conditions (specifically, assuming that the start time of pre-flight preparation cannot be shifted earlier than the standard 15 minute threshold) – $F^{\min}$	(1)	(2)–(5), (7)–(9), (13), where $p_i = 0$
2. An optimistic evaluation of the optimization criterion, based on a model solution using input data reflecting the most favorable operational conditions (specifically, allowing the start time of flight preparation to be shifted earlier than the standard 15 minute threshold, up to 360 minutes) – $F^{\max}$	(1)	(2)–(5), (7)–(9), (13), where p_i corresponds to the defined positive value
3. Maximization of the level of satisfaction – h	(12)	(2)–(5), (7)–(9), (13)–(16)
4. Maximization of the number of transported passengers at the maximum level of satisfaction – $\sum_{j\in I} c_j \cdot \sum_{\substack{i\in I\cup\{0\}\\ i\neq j}} x_{ij}$	(1)	(2)–(5), (7)–(9), (13)–(14), the value of h in (13) and (14) corresponds to the value of (12) calculated in Phase 3
5. Minimization of unproductive positioning flights for the maximum number of passengers at the maximum level of satisfaction – $\sum_{i\in I\cup\{0\}} \sum_{\substack{j\in I\cup\{0\}\\ j\neq i}} \tau_{ij} \cdot x_{ij}$	(17)	(2)–(5), (7)–(9), (13)–(14), (18), the value of h in (13) and (14) corresponds to the value of (12) calculated in Phase 3
6. Minimization of time shifts for the maximum number of passengers at a given level of satisfaction, while minimizing the number of unproductive positioning flights – $\sum_{i\in I} z_i$	(19)	(2)–(5), (8)–(9), (13)–(14), (18), (20)

Tab. I *Phases of the optimization approach.*

5. Computational Experiments

The calculations using the proposed models were conducted with partially real operational data provided by an existing airline operating scheduled international passenger flights over short and medium distances. The airline operates a fleet of approximately 70 aircraft. The fleet is homogeneous in terms of aircraft type, but heterogeneous with respect to aircraft capacity: the operator deploys both Airbus A320 aircraft with a capacity of 168 passengers and Airbus A321 aircraft with a capacity of 212 passengers. For the computational experiments, only flights operated by Airbus A321 aircraft were selected, which allows us to assume fleet homogeneity in terms of capacity as well.

The input data used in the model were derived from real-world operations and include:

– the airline's flight schedule, specifying planned departure and arrival times of individual flights,

– the preparation times required for each flight,

– the unproductive transfer times (ferry flights) between airports,

– the information on airports where time slots are assigned for departures and arrivals,

– the composition of the aircraft fleet.

The flight schedule corresponds to a real operational base of the airline on a specific day when 36 flights are scheduled to be operated by aircraft stationed at that base. Basic information about the flight schedule is presented in Tab. II, and unproductive transfer times between airports are detailed in Tab. III. In Tab. III, the values $M1$ and $M2$ represents infeasible unproductive transfers (positioning flights). Two types of infeasible transfers are distinguished:

– a transfer is infeasible if the start time of pre-flight preparation would occur before the aircraft's arrival at the departure airport ($M1$),

– a transfer is also considered infeasible if its duration exceeds 60 minutes, as such transfers are not performed due to operational unprofitability ($M2$).

The parameter subject to uncertainty in the optimization experiments was the maximum allowable shift in the start time of pre-flight preparation, where the standard value of 15 minutes was extended up to 360 minutes. Additionally, experiments are carried out at airports considered regulated in terms of operational planning, where the maximum permitted shift in scheduled times is limited to 15 minutes in accordance with the permitted application window for the allocated slot by air traffic control. The pre-flight preparation times are advanced relative to scheduled departure times by 30 minutes, a value consistent with typical turnaround times for short- and medium-haul flights. The durations of unproductive transfers (positioning flights) were determined in accordance with standard operational practice. An unproductive transfer is allowed if its duration does not

flight i	Plane	DEP	DEST	Plan DEP	Plan ARR	Act. DEP	Act. ARR	t_i	T_i	a_i	c_i
1	**DGA**	**ATH**	**MXP**	**4:40**	**7:15**	**4:58**	**7:26**	**250**	**155**	**15**	**158**
2	NAE	ATH	LCA	4:45	6:20	4:41	6:07	255	95	15	161
3	**NAF**	**ATH**	**LHR**	**5:15**	**9:15**	**5:36**	**9:36**	**285**	**240**	**15**	**88**
4	DNH	ATH	ZRH	5:30	8:20	5:30	8:27	300	170	15	142
5	**NAB**	**ATH**	**MAD**	**5:55**	**9:45**	**6:02**	**10:02**	**325**	**230**	**15**	**194**
6	NAC	ATH	ARN	6:00	9:45	6:10	9:46	330	225	15	179
7	NAM	ATH	HEL	6:25	10:10	6:36	10:19	355	225	15	156
8	DVZ	ATH	HER	7:00	7:55	7:16	8:06	390	55	15	195
9	NAE	LCA	ATH	7:15	9:00	7:11	9:02	405	105	15	142
10	**NAD**	**ATH**	**CDG**	**7:40**	**11:10**	**7:38**	**11:04**	**430**	**210**	**15**	**195**
11	DGA	MXP	ATH	8:10	10:35	8:42	11:00	460	145	15	118
12	DVZ	HER	ATH	8:35	9:25	8:48	9:43	485	50	15	177
13	DNH	ZRH	ATH	9:15	11:50	9:21	11:46	525	155	15	144
14	NAE	ATH	LCA	10:10	11:50	10:11	11:48	580	100	15	149
15	NAF	LHR	ATH	10:15	13:50	10:35	13:55	585	215	15	137
16	DVZ	ATH	HER	10:25	11:20	10:45	11:34	595	55	15	196
17	NAC	ARN	ATH	10:40	14:15	10:46	14:13	610	215	15	175
18	NAB	MAD	ATH	10:45	14:05	10:56	14:09	615	200	15	186
19	NAM	HEL	ATH	11:00	14:45	11:05	14:41	630	225	15	131
20	DVZ	HER	ATH	12:00	12:50	12:11	12:57	690	50	15	175
21	NAD	CDG	ATH	12:05	15:15	12:07	15:06	695	190	15	205
22	NAE	LCA	ATH	12:45	14:30	12:49	14:35	735	105	15	167
23	DNH	ATH	WAW	13:20	16:00	14:07	16:39	770	160	15	199
24	**DGA**	**ATH**	**MXP**	**13:50**	**16:25**	**13:53**	**16:32**	**800**	**155**	**15**	**179**
25	DVZ	ATH	HER	13:50	14:45	14:04	14:53	800	55	15	174
26	**NAF**	**ATH**	**LHR**	**15:15**	**19:10**	**15:24**	**19:13**	**885**	**235**	**15**	**110**
27	DVZ	HER	ATH	15:25	16:15	15:26	16:11	895	50	15	139
28	NAE	ATH	LCA	15:35	17:10	15:49	17:10	905	95	15	170
29	**NAM**	**ATH**	**CDG**	**15:50**	**19:20**	**16:01**	**19:30**	**920**	**210**	**15**	**199**
30	DNH	WAW	ATH	16:45	19:10	17:43	20:07	975	145	15	115
31	NAC	ATH	HER	17:15	18:05	17:34	18:24	1 005	50	15	211
32	DGA	MXP	ATH	17:20	19:45	17:21	19:35	1 010	145	15	114
33	NAE	LCA	ATH	18:00	19:45	18:04	19:50	1 050	105	15	122
34	NAC	HER	ATH	18:45	19:35	19:08	19:58	1 095	50	15	176
35	NAF	LHR	ATH	20:15	23:50	20:11	23:31	1 185	215	15	137
36	NAM	CDG	ATH	20:15	23:20	20:20	23:31	1 185	185	15	185

Tab. II *Used flights schedule (slot regulated flights are bold).*

		ti	Ti	ti+Ti	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
					ATH-LCA	LHR-ATH	ATH-HER	ARN-ATH	MAD-ATH	HEL-ATH	HER-ATH	CDG-ATH	LCA-ATH	ATH-WAW	ATH-MXP	ATH-HER	ATH-LHR	HER-ATH	ATH-LCA	ATH-CDG	WAW-ATH	ATH-HER	MXP-ATH	LCA-ATH	HER-ATH
					580	585	595	610	615	630	690	695	735	770	800	800	885	895	905	920	975	1005	1010	1050	1095
8	ATH-HER	390	55	445	50	M1	50	M1	M1	M1	0	M2	M2	50	50	50	50	0	50	50	M2	50	M2	M2	M2
9	LCA-ATH	405	105	510	0	M1	0	M1	M1	M1	55	M1	M2	0	0	0	0	55	0	0	M2	0	M2	M2	55
10	ATH-CDG	430	210	640	M1	M1	M1	M1	M1	M1	M1	0	M1	M1	M1	M1	M2	M2	M2	M2	M2	M2	M2	M2	M2
11	MXP-ATH	460	145	605	0	M1	0	M1	M1	M1	55	M1	M2	0	0	0	0	55	0	0	M2	0	M2	M2	55
12	HER-ATH	485	50	535	0	M1	0	M1	M1	M1	55	M1	M2	0	0	0	0	55	0	0	M2	0	M2	M2	55
13	ZRH-ATH	525	155	680	0	M1	0	M1	M1	M1	M1	M1	M1	0	0	0	0	55	0	0	M2	0	M2	M2	55
14	ATH-LCA	580	100	680	M1	M1	M1	M1	M1	M1	M1	M1	0	M1	M2	M2	M2	M2	M2	M2	M2	M2	M2	0	M2
15	LHR-ATH	585	215	800	M1	M1	0	M1	M1	M1	M1	M1	M1	0	0	0	0	55	0	0	M2	0	M2	M2	55
16	ATH-HER	595	55	650	M1	M1	M1	M1	M1	M1	0	M1	M1	50	50	50	50	0	50	50	M2	50	M2	M2	0
17	ARN-ATH	610	215	825	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	0	0	0	55	0	0	M1	0	M2	M2	55
18	MAD-ATH	615	200	815	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	0	0	0	55	0	0	M1	0	M2	M2	55
19	HEL-ATH	630	225	855	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	0	0	55	0	0	M1	0	M2	M2	55
20	HER-ATH	690	50	740	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	0	0	55	0	0	M1	0	M2	M2	55
21	CDG-ATH	695	190	885	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	0	0	0	55	0	0	M1	0	M2	M2	55
22	LCA-ATH	735	105	840	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	0	0	0	M1	0	0	M1	0	M2	M2	55
23	ATH-WAW	770	160	930	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	M1	M1	M1	M2
24	ATH-MXP	800	155	955	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	M1	M1
25	ATH-HER	800	55	855	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	50	50	M1	50	M1	M2	0
26	ATH-LHR	885	235	1120	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1
27	HER-ATH	895	50	945	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	0	M1	0	M1	M2	55
28	ATH-LCA	905	95	1000	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	M1
29	ATH-CDG	920	210	1130	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1
30	WAW-ATH	975	145	1120	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	M1	0	M2	M1	55

Tab. III *Part of the unproductive (positioning) flights matrix.*

exceed 60 minutes. For these transfers, airport slot constraints are not considered in the computational experiments. Slot restrictions apply to the following flights (Tab. III) 1, 3, 5, 10, 24, 26, 29. In the computational experiments, it is further assumed that the irregular operation (IRROP) is caused by a technical malfunction affecting aircraft in the fleet. The fleet located at the operational base comprises $N = 9$ aircraft, which are considered homogeneous, i.e., interchangeable and capable of operating any flight scheduled for that operational day. The malfunction is assumed to occur at the start of the operational day and is not repairable within that day, rendering the affected aircraft unavailable for the entire day. The experiments were conducted for various combinations of input parameters, including the number of available aircraft. The number of available aircraft on a given day was varied between $N = 9$ (all aircraft operational, no IRROP) and $N = 1$ (only one aircraft operational). The maximum allowed shift in the start times of flight preparation was set to 360 minutes, representing a commonly considered maximum operational delay in airline scheduling and from this value the delay is considered unacceptable.

6. Discussion of the Achieved Results

In general, it can be expected that allowing greater delays in the start times of pre-flight preparation creates more favorable conditions for improving the values of the optimization criteria – namely, the level of satisfaction and the total number of transported passengers.

However, achieving the maximum level of satisfaction does not automatically correspond to achieving the maximum number of transported passengers. This is evident in Tab. VI – with an available fleet size of $N-4$, the number of transported passengers in Phase 3 (in which the level of satisfaction is maximized) reaches 4648, while in Phase 4 (where the number of transported passengers is maximized under the level of satisfaction attained in Phase 3), this number increases to 4726. Thus, the inclusion of Phase 4 is justified.

The requirement to minimize the total duration of non-productive repositioning flights can also lead to an increase in the total amount of pre-flight preparation start-time shifts. Examples of such behavior are shown in Tab. VII, particularly in $N-0$, $N-5$ and $N-6$ during Phase 5. Since any deviation from the original

Phase	$N-0$	$N-1$	$N-2$	$N-3$	$N-4$	$N-5$	$N-6$	$N-7$	$N-8$
1	5800	5575	5299	5012	4412	3811	3138	2446	1443
2	5800	5575	5299	5013	4726	4125	3452	2668	1443
3	1	1	1	0.916	0.722	0.722	0.722	0.652	1
4	5800	5575	5299	5013	4726	4125	3432	2668	1443
5	0	0	0	0	0	0	0	0	0
6	0	0	15	70	300	315	275	375	0

Tab. IV *The values of the optimization criteria in the individual phases of the optimization calculation.*

Phase	$N-0$	$N-1$	$N-2$	$N-3$	$N-4$	$N-5$	$N-6$	$N-7$	$N-8$
1	5800	5575	5299	5012	4412	3811	3138	2446	1443
2	5800	5575	5299	5013	4726	4125	3452	2668	1443
3	5800	5575	5299	5013	4648	4125	3432	2668	1443
4	5800	5575	5299	5013	4726	4125	3432	2668	1443
5	5800	5575	5299	5013	4726	4125	3432	2668	1443
6	5800	5575	5299	5013	4726	4125	3432	2668	1443

Tab. V *The evolution of the values of the total number of passengers carried, achieved in the individual phases of the optimization calculation.*

Phase	$N-0$	$N-1$	$N-2$	$N-3$	$N-4$	$N-5$	$N-6$	$N-7$	$N-8$
1	210	210	105	105	105	0	0	0	0
2	105	210	105	105	0	0	0	0	0
3	0	0	105	105	105	0	0	0	0
4	315	105	210	105	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0

Tab. VI *The evolution of the values of total unproductive flight times achieved in the individual phases of the optimization calculation.*

Phase	$N-0$	$N-1$	$N-2$	$N-3$	$N-4$	$N-5$	$N-6$	$N-7$	$N-8$
1	55	55	45	30	30	15	0	0	0
2	250	145	890	320	835	1065	830	580	0
3	40	60	60	125	470	430	450	495	15
4	15	55	45	140	530	410	435	520	0
5	25	25	15	85	470	505	450	520	0
6	0	0	15	70	300	315	275	375	0

Tab. VII *Total time shifts in pre-flight preparation start times expressed in minutes achieved in individual steps of the optimization calculation.*

flight schedule – here represented by delayed pre-flight preparation—may affect passenger satisfaction with the service offering, it is desirable to minimize the total duration of such shifts. Therefore, the inclusion of Phase 6 is also well justified.

Special attention should be paid to the results corresponding to the available fleet sizes $N-0$ and $N-8$. In the case of $N-0$ (i.e., no irregular operations or IRROPs occur), the level of satisfaction of schedule realization is expected to equal 1.0, as the number of aircraft based at the airport is sufficient to execute the planned flight schedule for the operational day in question. Furthermore, both the total duration of non-productive repositioning flights and the total duration of pre-flight preparation start-time shifts are expected to be zero, as such adjustments are typically not present within a regular schedule. All three expected properties were confirmed by the computational experiments. In the case of $N-8$ (i.e., only one aircraft remains available), the level of satisfaction also reached a value of 1.0. Additionally, the total number of passengers served was 1443, with zero total duration of both non-productive repositioning flights and pre-flight preparation time shifts. This result is most likely explained by the availability of a single aircraft rotation that corresponds to the itinerary with the highest passenger count.

7. Conclusion

The increasing complexity and sensitivity of airline operations necessitate not only robust reactive and proactive systems, but also, and perhaps most critically, advanced predictive approaches to managing irregular operations (IRROPs). This paper has introduced a novel optimization model designed to address IRROPs with a primary focus on minimizing passenger impact—an aspect frequently overlooked in existing literature.

By incorporating fuzzy linear programming into the operational decision-making process, the model enables effective balancing of conflicting objectives: adherence to the flight schedule, resource utilization, and passenger satisfaction. The model accounts for uncertainty in key operational parameters, such as the start time of pre-flight preparation, and establishes a structured optimization process based on the attained level of schedule satisfaction.

This approach provides dispatchers with a quantifiable tool to prioritize the passenger-centric consequences of operational decisions under IRROP conditions, thereby contributing to improved service quality and strengthening the airline's competitive position. The multi-stage optimization procedure—initially maximizing the number of transported passengers, followed by the minimization of non-productive repositioning flights and temporal shifts—enhances the robustness and practical applicability of the proposed model.

Future research could further extend this framework by incorporating additional fuzzified parameters and exploring multi-criteria optimization methods that take into account financial costs, environmental impacts, or real-time crew availability. The integration of advanced optimization techniques and fuzzy logic into IRROP management represents a promising direction for achieving more resilient and passenger-oriented operational planning.

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