

FREQUENCY-BIN GEOMETRIC ALGEBRA FOR STRUCTURED AND RELATIONAL INFORMATION: THEORETICAL FOUNDATIONS

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Abstract: Frequency-Bin Geometric Algebra (FB-GA) is developed as a geometric-algebraic description whose coefficients are carried by selected spectral addresses. The construction keeps the algebraic meaning of a blade separate from the physical mechanism that produces or transports a frequency component. Starting from vector channels with complex quadrature coefficients, the paper derives signal signatures for oriented bivectors, trivectors obtained by exterior composition, and recursively defined higher grades. The carrier layer is positioned within classical signal theory by relating FB-GA to Fourier and time–frequency representations. Sparse support operations are used to identify shared blade structures without conflating them with new geometric components generated by explicit algebraic composition. The framework is then extended to multiple locations or subsystems by embedding their local vector spaces in a common relational algebra, allowing cross-location bivectors, trivectors, and higher-grade relational structures to be represented explicitly. The final discussion examines correlated distributed relations, the consolidation and expansion of multivector structure, and the possibility that FB-GA systems may evolve dynamically through the formation, reinforcement, and reorganization of relational patterns.

Key words: *geometric algebra, frequency-bin encoding, multivectors, bivectors, trivectors, relational information, distributed objects, higher grades*

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1. Introduction

Discrete frequency modes are widely used as controllable information channels in photonic and radio-frequency systems [1, 2]. Geometric algebra (GA), on the other hand, organizes scalars, directed components, oriented planes, volumes, and their higher-dimensional extensions within an algebraic language [3, 4]. FB-GA brings these viewpoints together by using frequency channels as carriers for coefficients of explicitly labelled GA blades.

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Classical signal and communication theory provides the natural signal-domain foundation for this construction. Shannon's framework formalized information transmission through channels subject to bandwidth and noise constraints [13]. Gabor introduced a joint time–frequency viewpoint in which the localized signal content is represented by shifted and modulated elementary functions [14]. Coherent digital communication subsequently developed two-dimensional amplitude phase signal spaces and quadrature modulation as practical ways to assign information to orthogonal carrier components [15, 16]. Wavelet and multiresolution theory added localized, scale-dependent decompositions and exact-reconstruction filter-bank structures [17–19]. FB-GA uses these established signal representations as possible carrier mechanisms; its additional contribution is the explicit assignment of geometric-algebra grade, blade identity, and orientation to selected coefficients and relations.

The distinction is between *where a coefficient is carried* and *what the coefficient represents*. A spectral component acquires a vector, bivector, or higher-grade meaning only through a stated algebraic model and an associated coding convention. Consequently, ordinary signal multiplication, which can create sum and difference frequencies, must not be identified with the antisymmetric exterior product. FB-GA uses signal processing to realize or observe candidate channels, while the GA layer specifies blade identity, grade, and orientation.

This separation is useful in problems where relations among components are as important as the components themselves. It permits a sparse object to contain elementary features together with explicitly represented pairwise and higher-order relations, and it also allows such structures to be distributed over different locations or subsystems. These ideas are relevant to frequency-bin information processing, higher-order spectral organization, and prospective neural or cognitive representations [5–8].

Importantly, FB-GA is not limited to assigning spectral addresses to vectors, bivectors, trivectors, and higher-grade blades separately. The algebra also provides a structured representation of relations among objects of different grades. Consequently, vector–bivector, vector–trivector, bivector–trivector, and more general multigrade couplings can themselves carry information. Such inter-grade relations provide an additional representational layer beyond the spectral modulation of individual coefficients.

The present paper concentrates on the mathematical and representational foundations: the construction of frequency-addressed vectors and blades, exterior formation of higher grades, sparse support operations, and relational algebras for distributed objects.

2. Geometric-algebra preliminaries

Let V be an m-dimensional Euclidean vector space with orthonormal basis $\{e_1, \ldots, e_m\}$. For vectors $a, b \in V$, the geometric product is

$$ab = a \cdot b + a \wedge b. \tag{1}$$

Here, $a \cdot b$ is the scalar inner product and $a \wedge b$ is the exterior product. Eq. (1) separates metric similarity, represented by the scalar part, from oriented independence, represented by the exterior part.

For distinct orthogonal basis vectors,

$$e_i e_j = e_i \wedge e_j = e_{ij}, \qquad e_j e_i = -e_{ij}. \tag{2}$$

The blade e_{ij} represents an oriented plane. Reversing the order of the two basis directions reverses the orientation and therefore changes the sign.

For three distinct basis directions,

$$e_{ijk} = e_i \wedge e_j \wedge e_k \tag{3}$$

represents an oriented grade-three element. The word *grade* denotes the number of independent basis directions contained in the blade: vectors have grade one, bivectors grade two, and trivectors grade three.

A general multivector is

$$M = \sum_{A \subseteq \{1,\dots,m\}} c_A e_A, \tag{4}$$

where A is an ordered index set, e_A is the corresponding blade, $e_\varnothing = 1$, and $c_A \in \mathbb{R}$ or $c_A \in \mathbb{C}$ is its coefficient. The coefficient magnitude describes the strength of the labelled component. A complex phase may carry a common timing reference, an orientation code, or other encoder metadata. There are 2^m basis blades, including the scalar. FB-GA is primarily designed for sparse objects, for which only $K \ll 2^m$ coefficients are active.

3. Frequency-bin representation of vectors, bivectors, trivectors, and higher grades

The construction proceeds through three conceptually distinct layers: carrier addresses, GA blade semantics, and relations among the resulting blades. The first specifies where a coefficient is represented, the second specifies its geometric meaning, and the third specifies how the represented geometric objects are organized with respect to one another.

3.1 Grade-one channels and quadrature coefficients

Let the basis vector e_i be assigned a carrier address f_i. A real grade-one channel is represented by two quadrature envelopes,

$$s_i(t) = I_i(t)\cos(2\pi f_i t) + Q_i(t)\sin(2\pi f_i t). \tag{5}$$

The functions $I_i(t)$ and $Q_i(t)$ are the in-phase and quadrature envelopes of a channel. They jointly specify the real signal carried out at the address f_i.

Using the convention

$$C_i(t) = I_i(t) - \mathrm{i}Q_i(t), \tag{6}$$

Eq. (5) becomes

$$s_i(t) = \mathrm{Re}\left\{C_i(t)\mathrm{e}^{\mathrm{i}2\pi f_i t}\right\}. \tag{7}$$

The magnitude $|C_i(t)|$ is the coefficient strength and $\arg C_i(t)$ is its phase relative to the selected reference. The carrier f_i is a spectral address.

For a sparse vector object with an active index set $\mathcal{I}_1$,

$$s_V(t) = \sum_{i\in\mathcal{I}_1} \mathrm{Re}\left\{C_i(t)\mathrm{e}^{\mathrm{i}2\pi f_i t}\right\}. \tag{8}$$

Eq. (8) is a linear superposition. It places several labelled vector channels in one waveform but does not itself create a bivector or any other higher grade.

The I/Q representation in Eqs. (5)–(7) is likewise standard in coherent modulation. Two orthogonal carrier components provide a two-dimensional signal space in which amplitude and phase can be jointly controlled [15, 16]. FB-GA does not redefine this modulation principle. Instead, it uses the complex coefficient as the numerical content of a labelled GA component and keeps the blade semantics separate from the physical modulator.

3.2 Bivector coefficients and paired signature

Consider two real channels

$$s_i(t) = I_i(t)\cos\omega_i t + Q_i(t)\sin\omega_i t, \tag{9}$$

$$s_j(t) = I_j(t)\cos\omega_j t + Q_j(t)\sin\omega_j t. \tag{10}$$

Their ordinary product is

$$\begin{aligned} s_i(t)s_j(t) = {} & \frac{1}{2}(I_i(t)I_j(t) + Q_i(t)Q_j(t))\cos[(\omega_i-\omega_j)t] \\ & + \frac{1}{2}(Q_i(t)I_j(t) - I_i(t)Q_j(t))\sin[(\omega_i-\omega_j)t] \\ & + \frac{1}{2}(I_i(t)I_j(t) - Q_i(t)Q_j(t))\cos[(\omega_i+\omega_j)t] \\ & + \frac{1}{2}(I_i(t)Q_j(t) + Q_i(t)I_j(t))\sin[(\omega_i+\omega_j)t]. \end{aligned} \tag{11}$$

Equivalently,

$$s_i(t)s_j(t) = \frac{1}{2}\,\mathrm{Re}\left\{C_i(t)C_j^*(t)\mathrm{e}^{\mathrm{i}(\omega_i-\omega_j)t} + C_i(t)C_j(t)\mathrm{e}^{\mathrm{i}(\omega_i+\omega_j)t}\right\}. \tag{12}$$

The asterisk denotes complex conjugation. The first term carries the difference of the parent phases, while the second carries their sum. These equations are signal-processing identities. They explain why mixers and nonlinear systems generate sum and difference frequencies. These frequencies provide spectral addresses; they do not by themselves constitute the bivector.

However, Eq. (11) is symmetric under exchange of the two real input signals. It therefore does not by itself implement

$$e_i \wedge e_j = -e_j \wedge e_i. \tag{13}$$

FB-GA adds a separate algebraic orientation convention. For a canonical order, define

$$\eta_{ij} = \begin{cases} +1, & i < j, \\ -1, & i > j. \end{cases} \tag{14}$$

The factor η_{ij} is implemented as a coefficient sign or an additional phase shift of π.

A bivector is an oriented relation between independent basis directions e_i and e_j. Its algebraic form is

$$B_{ij}(t)e_{ij} = B_{ij}(t)e_i \wedge e_j, \qquad i < j. \tag{15}$$

The real or complex coefficient $B_{ij}(t)$ may be assigned by an encoder, obtained from a physical model, or estimated from data.

A paired real frequency-bin signature can be written as

$$s_{ij}^{(2)}(t) = \eta_{ij}\Big[\operatorname{Re}\{B_{ij}^{(-)}(t)\mathrm{e}^{\mathrm{i}2\pi|f_i-f_j|t}\} + \operatorname{Re}\{B_{ij}^{(+)}(t)\mathrm{e}^{\mathrm{i}2\pi(f_i+f_j)t}\}\Big]. \tag{16}$$

The coefficients $B_{ij}^{(-)}(t)$ and $B_{ij}^{(+)}(t)$ describe the complex amplitude and phase associated with the difference- and sum-frequency channels, respectively. For ordinary physical mixing, they follow directly from Eq. (12). Using

$$C_k(t) = I_k(t) - \mathrm{i}Q_k(t), \tag{17}$$

they are

$$B_{ij}^{(-)}(t) = \frac{1}{2}C_i(t)C_j^*(t), \tag{18}$$

$$B_{ij}^{(+)}(t) = \frac{1}{2}C_i(t)C_j(t). \tag{19}$$

Equivalently, in terms of the quadrature components,

$$B_{ij}^{(-)}(t) = \frac{1}{2}\Big[I_i(t)I_j(t) + Q_i(t)Q_j(t) + \mathrm{i}(I_i(t)Q_j(t) - Q_i(t)I_j(t))\Big], \tag{20}$$

$$B_{ij}^{(+)}(t) = \frac{1}{2}\Big[I_i(t)I_j(t) - Q_i(t)Q_j(t) - \mathrm{i}(I_i(t)Q_j(t) + Q_i(t)I_j(t))\Big]. \tag{21}$$

The two channels generally carry different phase information, and may also differ in amplitude, because the difference-frequency term contains the complex conjugate $C_j^*(t)$, whereas the sum-frequency term contains $C_j(t)$. In an FB-GA encoder, however, these channels can be deliberately assigned as two spectral representations of the same bivector coefficient $B_{ij}(t)$.

3.3 Bivector construction and reconstruction

For the active bivector index set $\mathcal{I}_2 = \{(i,j) : i < j,\ B_{ij}(t) \neq 0\}$, the complete grade-two part of the waveform is

$$s_B(t) = \sum_{(i,j)\in\mathcal{I}_2} \eta_{ij}\Big[\operatorname{Re}\{B_{ij}^{(-)}(t)\mathrm{e}^{\mathrm{i}2\pi|f_i-f_j|t}\} + \operatorname{Re}\{B_{ij}^{(+)}(t)\mathrm{e}^{\mathrm{i}2\pi(f_i+f_j)t}\}\Big]. \tag{22}$$

Thus $s_B(t)$ is the superposition of all active paired bivector signatures.

For real-valued signals, one possible construction of the bivector coefficient is based on their analytic-signal representations. Let $x_i(t)$ denote the measured or preprocessed signal assigned to the basis direction e_i. Its analytic representation is

$$z_i(t) = x_i(t) + \mathrm{i}\mathcal{H}\{x_i(t)\}, \tag{23}$$

where $\mathcal{H}$ denotes the Hilbert transform that provides a quadrature companion shifted by approximately 90° relative to the real signal, linking the coefficient representation to the classical analytic-signal and time–frequency viewpoint [14].

An antisymmetric bivector coefficient can be defined as

$$B_{ij}(t) = \mathrm{Im}\{z_i(t)z_j^*(t)\}, \qquad B_{ji}(t) = -B_{ij}(t). \tag{24}$$

During decoding, the known channel-dependent gains and phase shifts are compensated, so that both corrected channels should yield approximately the same estimate of the underlying bivector coefficient. The consistency condition is therefore

$$\widehat{B}_{ij}^{(-)}(t) \approx \widehat{B}_{ij}^{(+)}(t) \approx B_{ij}(t). \tag{25}$$

The agreement of the two reconstructed coefficients is therefore part of the decoding rule for identifying the paired spectral signature as one bivector.

3.4 Trivector construction by exterior composition

A grade-three blade is created by explicitly extending an oriented plane in an independent direction. Let $B_{ij}(t)e_{ij}$ be an identified bivector and let $C_k(t)e_k$ be a vector channel with $k \notin \{i, j\}$. Their exterior composition is

$$\big(B_{ij}(t)e_{ij}\big) \wedge \big(C_k(t)e_k\big) = \eta_{ijk}B_{ij}(t)C_k(t)e_{ijk}. \tag{26}$$

Here $\eta_{ijk} \in \{-1, +1\}$ is the sign of the permutation required to transform the ordered indices (i, j, k) into canonical increasing order. The grade-three coefficient is therefore defined hierarchically as

$$T_{ijk}(t) = \eta_{ijk}B_{ij}(t)C_k(t). \tag{27}$$

A grade-three element is created geometrically by extending the already oriented bivector plane e_{ij} with a new independent direction e_k. If the added direction coincides with one of the directions already contained in the bivector,

$$e_{ij} \wedge e_k = 0 \quad \text{for} \quad k \in \{i, j\}, \tag{28}$$

so no trivector is formed. This follows directly from the antisymmetry of the exterior product: a new grade can arise only when the added direction contributes an independent geometric degree of freedom.

The same blade can also be composed directly from three vectors,

$$(C_i(t)e_i) \wedge (C_j(t)e_j) \wedge (C_k(t)e_k) = \eta_{ijk}C_i(t)C_j(t)C_k(t)e_{ijk}, \tag{29}$$

provided all three basis directions are distinct. Eq. (26) is preferable when the pair relation $B_{ij}(t)$ is already a meaningful object in its own right, because it explicitly expresses the hierarchy *vector* $\rightarrow$ *bivector* $\rightarrow$ *trivector*.

A trivector can be represented by four signed-combination addresses,

$$\mathcal{F}_{ijk} = \{f_i + f_j + f_k,\ f_i + f_j - f_k,\ f_i - f_j + f_k,\ -f_i + f_j + f_k\}. \tag{30}$$

Actual positive carrier values must be selected so that these addresses do not collide with vector or bivector channels. Let $f_{ijk,r}$ denote the rth member of a collision-free implementation of this address group. The real signature is

$$s^{(3)}_{ijk}(t) = \eta_{ijk} \sum_{r=1}^{4} \mathrm{Re}\{T_{ijk,r}(t) \mathrm{e}^{\mathrm{i}2\pi f_{ijk,r} t}\}\,. \tag{31}$$

For the active trivector index set $\mathcal{I}_3 = \{(i,j,k) : i < j < k,\ T_{ijk} \neq 0\}$, the complete grade-three contribution is

$$s_T(t) = \sum_{(i,j,k)\in\mathcal{I}_3} \eta_{ijk} \sum_{r=1}^{4} \mathrm{Re}\{T_{ijk,r}(t) \mathrm{e}^{\mathrm{i}2\pi f_{ijk,r} t}\}\,. \tag{32}$$

Thus, $s_T(t)$ collects all active four-channel trivector signatures in the same way that $s_V(t)$ and $s_B(t)$ collect vector and bivector channels.

3.5 One real FB-GA signal

The construction of the trivector described above can be continued by adding further independent basis directions. For example, a trivector can be extended to a grade-4 blade as

$$(T_{ijk}(t)e_{ijk}) \wedge (C_l(t)e_l) = \eta_{ijkl} T_{ijk}(t) C_l(t)\, e_{ijkl}, \qquad l \notin \{i,j,k\}. \tag{33}$$

The same rule can be applied recursively to construct still higher-grade blades. In FB-GA, each resulting blade is represented by its own predefined group of collision-free frequency addresses with the corresponding orientation and phase relations.

Combining all active grades gives

$$s_{\text{FB-GA}}(t) = s_V(t) + s_B(t) + s_T(t) + \cdots\,. \tag{34}$$

The physical signal is one-dimensional in time, but its labelled spectral structure represents a sparse multivector. This is analogous to a communication waveform carrying several logical channels: the dimension of the represented object is not the dimension of the wire or propagation medium.

3.6 Sparse support, intersection, and exterior extension

The support representation is useful not only for storing active blades sparsely, but also for exposing the relational topology of a multivector: which geometric components are present, shared, extended, or linked through higher-grade constructions.

The information represented by $s_{\text{FB-GA}}(t)$ therefore need not reside only in the coefficients of its individual grades. It may also reside in the organization among grades: for example, in which vector components participate in particular bivectors, which bivectors are extended into particular trivectors, or which higher-grade structures remain mutually coupled.

A sparse object can be stored as

$$\mathcal{M} = \{A \mapsto c_A : \ c_A \neq 0\}. \tag{35}$$

The label A identifies a blade, and c_A is its coefficient. The condition $c_A \neq 0$ means that the blade is active. The support

$$\operatorname{supp}(\mathcal{M}) = \{A : \ c_A \neq 0\} \tag{36}$$

contains the labels of all active blades.

For two sparse objects $\mathcal{A}$ and $\mathcal{B}$, their blade-wise intersection is

$$\mathcal{A} \cap_{\text{blade}} \mathcal{B} = \operatorname{supp}(\mathcal{A}) \cap \operatorname{supp}(\mathcal{B}). \tag{37}$$

This operation asks which labels occur in both objects. It is not the universal *GA meet*, which depends on the algebraic model and may require duality or projective structure. Similarly, a blade-wise union combines labels but does not automatically create a higher grade. A new grade is introduced only by an explicit exterior composition such as Eq. (26).

4. Spatially distributed and relational FB-GA objects

The preceding construction describes relations within one algebraic object; the same principle can be extended to relations among distinct objects or locations. FB-GA objects do not need to be located at one measurement point. Let V^A, V^B, and V^C denote local vector spaces associated with three positions, devices, subsystems, or observers. Their local multivectors satisfy

$$M^A \in \operatorname{Cl}(V^A), \qquad M^B \in \operatorname{Cl}(V^B), \qquad M^C \in \operatorname{Cl}(V^C). \tag{38}$$

$\operatorname{Cl}(V)$ denotes the geometric (Clifford) algebra generated by the vector space V, containing scalars, vectors, bivectors, trivectors, and their multivector combinations. Each local object may contain its own vectors, bivectors, trivectors, and higher grades. A superscript such as A identifies the location.

A common relational algebra can be defined over the direct sum,

$$\mathcal{A}_{\text{rel}} = \operatorname{Cl}(V^A \oplus V^B \oplus V^C). \tag{39}$$

The direct sum keeps the local basis directions distinct. Local spaces are treated as distinct copies within the enlarged coding space; this mathematical independence is a representational convention and does not imply physical orthogonality between

the corresponding locations. Consequently, a vector e_i^A at location A and a vector e_j^B at location B can form a relational bivector,

$$R_{ij}^{AB} = r_{ij}^{AB} e_i^A \wedge e_j^B. \tag{40}$$

The coefficient r_{ij}^{AB} represents a jointly defined relationship. In general, it cannot be inferred from M^A or M^B separately; it requires a common measurement, a shared phase convention, or an explicit encoder that relates the two locations.

A local bivector can be exterior-composed with a vector from another place,

$$T_{ij|k}^{AAB} = B_{ij}^A C_k^B \, e_i^A \wedge e_j^A \wedge e_k^B, \tag{41}$$

or a vector at A can extend a local bivector at B,

$$T_{i|jk}^{ABB} = C_i^A B_{jk}^B \, e_i^A \wedge e_j^B \wedge e_k^B. \tag{42}$$

The vertical bar in the indices separates the locations of the participating directions. These equations are hierarchical exterior compositions. They are not claims of physical causation.

More generally, blades associated with different locations or subsystems can be combined into higher-order relational structures, provided that their basis directions remain distinct within the common algebra. Such structures represent information that is defined only by the joint configuration of the participating subsystems and cannot, in general, be recovered from the local descriptions alone. For example, two local systems may exhibit nearly identical marginal spectra while differing in their cross-location phase relationships, polarization states, or higher-order coordination. A relational FB-GA representation can therefore reveal structure that remains hidden when each subsystem is analysed separately.

A broader conceptual step is an *autonomous FB-GA ecosystem*. Instead of treating multivectors as static encoded objects, one may allow interacting objects to modify their active supports and coefficients. New relations could be activated by exterior composition, persistent relations could be reinforced, and collective structures could span several objects. Such an extension would require explicit dynamical laws for activation, suppression, reinforcement. The proposal is therefore best viewed in the broader sense of information-physics, by using structures inspired by physics as formal models for complex information systems [11, 12].

5. Discussion

FB-GA extends frequency-domain representation beyond independently encoded spectral components by introducing explicit geometric and relational structure. In addition to individual vectors, bivectors, trivectors, and higher-grade blades, the framework can represent relations among objects of the same or different grades. These relations need not correspond to exterior composition: they may instead describe persistent amplitude dependencies, phase relations, relative orientations, shared temporal evolution, or other structured couplings. The resulting representation is therefore more naturally viewed as a *multigrade relational structure* than as a collection of modulated spectral channels.

This distinction opens a further level of description based on the collective organization of GA objects. Two FB-GA states may contain similar individual blades and even similar grade-wise coefficient distributions while differing substantially in how their components are coordinated. In such cases, the informative structure resides not only in which geometric objects are present, but also in the relational architecture connecting them.

The same principle extends naturally to distributed systems. By embedding location-specific vector spaces into a common algebra, FB-GA can represent cross-location structures whose information arises only from the joint configuration of several subsystems. This may be useful whenever local observations appear similar but differ in their phase relationships, orientation, polarization, or higher-order coordination. Such distributed relational descriptions are compatible with integrated information assessment [10].

A natural future extension is the transition from purely temporal frequency bins to spatial and spatiotemporal representations. Spatial modes may be characterized by wave vectors $\mathbf{k} = (k_x, k_y, k_z)$, while a joint representation $(\mathbf{k}, \omega)$ can describe structures evolving in space and time. Within this setting, vectors represent elementary modes, whereas bivectors and higher-grade components describe increasingly complex geometric relations among them. This perspective is particularly relevant to complex adaptive and neural systems, where information is distributed across locations, scales, frequencies, and temporal processes.

These ideas provide a natural conceptual basis for *Spatiotemporal Geometry Integration Theory* (SGIT). SGIT seeks to characterize integrated system states through the organization of distributed spatiotemporal geometric relations. Its future mathematical development may be formulated in terms of a *Spatiotemporal Multigrade Relational Geometry*, in which the informative object is not only a set of GA components, but the persistent organization linking them across space, time, and grade.

Future research can therefore investigate whether such multigrade relational patterns provide more robust state descriptors than individual blade coefficients alone. Potential application areas include distributed sensing, multimodal data fusion, adaptive networks, photonic and quantum-like information processing, biological systems, and neural and cognitive dynamics. In this broader view, FB-GA provides a foundation for studying how geometric objects and their relations collectively organize into persistent structures that characterize complex system states.

6. Conclusion

This paper establishes the theoretical foundations of Frequency-Bin Geometric Algebra (FB-GA) as a framework in which frequency channels carry coefficients while geometric-algebra labels specify their structural meaning. Vector, bivector, trivector, and higher-grade components are therefore not identified by spectral peaks alone, but by an explicit algebraic model that defines grade, orientation, and relational role.

A central distinction of FB-GA is the separation between physical signal generation and geometric composition. Nonlinear mixing may generate sum- and difference-frequency components, but such spectral products do not by themselves

constitute bivectors, trivectors, or higher-grade objects. Instead, they provide candidate carrier channels whose geometric interpretation must be supplied by the GA layer. This distinction prevents combination frequencies from being mistaken for higher-grade geometry and makes the representational assumptions explicit.

The resulting hierarchy follows directly from geometric algebra. Oriented vector pairs form bivectors, bivectors can be extended by new independent directions to form trivectors, and the same principle continues recursively to higher grades. Sparse support makes this hierarchy practical by retaining only the active geometric components of a given state.

The main conceptual extension of FB-GA is that information need not reside only in individual blade coefficients. It may also reside in the structured relations among blades of the same or different grades. FB-GA therefore represents not merely a collection of modulated spectral components, but a multigrade relational architecture in which vectors, bivectors, trivectors, and higher-grade structures can participate in persistent and coordinated patterns.

This perspective extends naturally to distributed systems. By embedding location-specific vector spaces into a common relational algebra, FB-GA can represent cross-location structures whose information exists only in the joint configuration of several subsystems. It offers a common language for encoding local components, higher-order geometric structure, and collective relational organization within one sparse multivector framework. This opens the way toward dynamical extensions in which relations can emerge, persist, reorganize, and form larger coordinated structures. Practical encoding, extraction, and application scenarios are developed in the companion paper [20], while future work will investigate how multigrade relational organization can be used to characterize distributed and integrated spatiotemporal systems.

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